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ABSTRACT

Mathematics and the use of mathematical thinking should be much more than what has been traditional school arithmetic. Much of the mathematical reasoning can be developed and experienced out of school, particularly in the home. This material is a teacher's guide designed to help parents support what is done with their children in class. Background material for parents is provided. Some underlying principles in teaching mathematics meaningfully are listed. Assessment record sheets are provided. A total of 61 lesson plans covering the following topics are included: (1) number concept; (2) number operation; (3) place value; (4) problem solving; and (5) measurement. (YP)

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# MEANINGFUL MATHEMATICS

# LEVEL ONE

## TEACHER'S GUIDE TO LESSON PLANS

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## *MEANINGFUL MATHEMATICS*

## *TEACHER'S GUIDE FOR LEVEL ONE*

## *INTRODUCTION*

Mathematical activities at this level are to include a continuation of classifying, comparing, ordering and patterning; development of number concept for each number in order from two through as far as each individual child can progress; development of understanding of place value through use of small numbers for grouping; emphasis on the situations that give rise to adding, subtracting, multiplying and dividing and hence, when to use each of these; further development of understanding of equality and the symbol for that; further development of geometry understanding; an emphasis on thinking; understanding of the meaning of the symbols used with arithmetic operations.

The year should begin with an assessment of each child's knowledge in these areas and the beginning of each child's assessment record. These assessments should be performed at least two more times during the year.

No classification or ordering lessons are included in this guide. Basically, the same activities can be done as in kindergarten, with emphasis on the extensions in those lessons. ***Classifying*** can move toward classifying into more than two groups, classifying using joint presence of two or more properties, and the use of multiple classification, ie. red AND triangle.

Teachers at this level should have the ***Kindergarten Guide*** and re-do original lessons found there, or modify them for use at **Level One**, in the areas of ordering and patterning.

***Number Concept*** development along with ***Classifying, Ordering, Comparing, Joining, Separating*** and ***Patterning Activity***, is likely to dominate the early part of the year. As enough students indicate knowledge of "four", ***Place Value Development*** should start along with ***Number Operation*** work. All of this activity should be continuous throughout the year, so some of the lessons can be repeated as needed. Try to stay with some concepts until reasonable mastery. An example is ***Equality*** early in the year. Another is ***Separating*** as this relates to subtraction and division.

By the end of this year, children MUST master ***Number*** through ten; ***Place Value*** as this involves grouping when a given amount is accrued, decomposing groups when needed, and positional placement of digits to

decomposing groups when needed, and positional placement of digits to indicate WHAT is counted by each; knowledge of what situations require which arithmetic operations; and meanings of symbols. ***Thinking*** and ***Problem Solving*** is the common thread running through ALL activity and every situation that provides opportunity for problem solving must be exploited.

While emphasis is on mastery of each concept, it is a good idea to periodically build in an activity that reviews all concepts and skills previously learned. Once each week is a good schedule for elementary school. This can be done orally in the primary grades and with a written exercise after symbolic representations have been mastered. Since all operations arise from combining, separating, comparing and part whole relations, as soon as the operation symbols are understood, these operations should all be presented at the same time in the written review exercises.

It is easier to do this at the beginning levels with orally presented review activities.

It is axiomatic for good teaching that past learnings should ALWAYS be integrated into new learning activities.

KINDERGARTEN AND LEVEL ONE

## SOME UNDERLYING PRINCIPLES IN TEACHING MATHEMATICS MEANINGFULLY

1. Keep a proper balance between teaching and the fostering of spontaneous growth.

    a. Provide the environment
    b. Respond to the children's interests
    c. Don't intervene too much
    d. Remember that children learn most, if not all, things in THEIR OWN WAY.

2. Get acquainted with children's intellectual life.

    a. Play is children's work
    b. Find out what they are trying to understand
    c. Be sensitive to their errors - they usually correct themselves
    d. Remember that their arithmetic may be different from yours

3. Make available to children a stimulating enviroment.

    a. Use the natural environment as much as possible
    b. Remember that things and events are not stimulating in themselves, only to the extent that they meet children's needs and intellectual concerns

4. Trust and support children's intellectual work.

    a. They are continually learning - find out what
    b. Permit children to think - encourage it

5. Give <u>special</u> support to children's counting.

    a. This is the area in which you are likely to do the most good
    b. Help them memorize counting words
    c. Help them see the rules for forming words for

larger numbers

d. Have them count objects in a variety of ways - touching, moving, one at a time, by two's, by three's, etc.
e. Give them simple addition, subtraction, multiplication, and division activities with real objects

6. Don't try to teach little children the terminology of "new math."

7. Avoid too much formal training, especially directed verbal instruction.

8. Don't bother with "readiness" activities.

   a. Children already know much
   b. Children's don't need to be made ready for math - they are. Engage them in it!

9. Don't waste time trying to train conservation behavior.

   a. It doesn't work
   b. Children learn to conserve in due course

10. Keep in mind the child's tremendous accomplishments.

    a. They already know much of what some programs are trying to "teach"
    b. Don't focus on weaknesses
    c. Mathematicis is easy to learn - engage them in the thinking that underlies mathematics concept formation

## WHAT TYPE OF ENVIRONMENT SHOULD THERE BE IN THE CLASSROOM SO THAT IT IS CONDUCIVE TO THE LEARNING PROCESS?

1. Children should be able to freely interact with their peers.
   a. Only through this interaction will the child be able to take all the different concepts and make these into one coherent whole.

2. Children should have the freedom to discover for themselves.
   a. How else could children develop THEIR OWN cognitive world unless they are allowed to act on objects and discover relationships between these objects?
   b. If a child is allowed to discover, through proper guidance from the teacher, lasting learning experiences result.

3. Children should be provided with activities that stimulate logico-mathematical reasoning.
   a. The children should have countless experiences that provide them with the opportunity to develop their own logic and reasoning powers.
   b. Every child should be encouraged to look to themselves for the correct answer, and this can come about only if they have frequent successes.

4. Children need individualized activities so that they can think and reason without outside interference.
   a. Opportunities should be made available for each child to work alone during some part of each day
   b. They should be presented with enough challenges to promote their intellectual development.

5. It is very difficult to differentiate between work and play when a child is acting on objects.
   a. Once a child knows that knowledge lasts forever; then and only then is lasting learning taking place

6. You, as a teacher, should be ever aware of how children learn.
   a. Children are continually taking in sensation, nourishment, ideas, and all sorts of information from the physical world. This taking-in process was called "assimilation" by Piaget. This assimilation is continuously balanced by "accommodation," which is the adjusting process of reaching out to the environment. These two processes function simultaneously at all biological and intellectual levels in both physical and intellectual development.
   b. A child is continually seeking to find a balance between the taking-in process and the adjusting process. This interplay between these two functions was labeled as

"equilibration" by Piaget. A child's mind seeks equilibrium between what he understands and what he experiences in his environment.

7. The teacher must teach each child to be able to function independently without supervision.

8. The classroom atmosphere should be such that there is a continual interaction between student and teacher, student and peers, student and environment.
   a. The teacher should talk TO the children and not AT the children
      1. This is so important - love and compassion for the children changes the whole classroom atmosphere completely
   b. Be yourself and make your mistakes with the children

9. Children should be allowed to make mistakes, for only through their mistakes does lasting learning take place.
   a. Never tell a child that he has the wrong answer. Ask questions so that you lead the child to discover a better answer for himself.
      1. If a child continually gives an answer that isn't the generally accepted one, then you, the teacher, should provide those kinds of activities that will eventually lead the child to see the error in his previous logic.

10. Every child should have a wide variety of experiences dealing with the same concept.
    a. Children should have many different experiences that deal with a given concept so that they expand their imagery of the concept.
    b. Children are constantly drawing on these past experiences and accommodating them to fit into their new experiences.
    c. Many of the experiences should be repeated over several times so that a child internalizes a concept. They know because it's in the constructed knowledge structure, not in short term rote memory.

GETTING STARTED:

1. Do individual assessments of each child to determine the following:

   a. How far can the child recite the whole numbers?
   b. What other oral counting skills does the child have - counting on? counting back? skip counting?
   c. How far in the number sequence has the child internalized numbers in the sense of recognizing invariance of number, knowing the part-part-whole relationship of number, identification of numerals as related to the number of property of a collection of things.
   d. Can the child write the numerals?
   e. Does the child know the most common meanings of "+", "-", "="?

2. Establish number stations with the materials available to you - Unifix Cubes, colored beans, Pattern Blocks, tiles, beads, toothpicks or matchsticks, Geoboards.

3. Prepare the recording forms for each kind of materials and for all numbers you expect the children to be working on during the first half of the year.

4. Have the children bring the materials from home for their "Treasure Chests."

5. Prepare the graphs you will use - Birthday cake and tooth, for example.

6. Look over Mathematics Their Way, Workjobs, Workjobs II, and Workjobs for Parents, if these are available to you. Of these, you should have a copy of Mathematics Their Way as a personal copy. Use this to interpret the purposes of the lessons, and to find extended activities to supplement the lessons.

7. Set up a schedule to make other things that you will need such as:
equality boards
place value mats
numeral and other symbol cards
overhead transparency materials

lap chalk boards
spinners
all purpose personal graphs
sorting bins or boxes
(see Appendix)

8. Obtain for group use non-commercial materials over and above those in the children's individual treasure chests. These might include: buttons of all kinds; old keys; washers; nuts and bolts; a variety of nuts or nutshells; old magazines and catalogs to cut up; samples of wallpaper; old Christmas cards and other materials that will lend themselves to classifying and ordering and counting activity.

9. The recommended materials for use in Grade One are:

a. Unifix Cubes - 500-1000 in <u>five</u> colors. (The boxes come with ten colors. These should be split into 2 sets of 5 colors for 2 teachers to use). Unifix are used for number, place value activity, patterns, counting and number operations, measurement, problem solving, and equality.

b. Pattern Blocks - 3 pails for a normal class. Use wood, rather than plastic. These are for use in patterns work for non-numeric problem solving and intuitive fraction and area work.

c. Geoboards - the large 12 x 12 geoboards are best for this grade. These are used for classifying, number concept, area and shape.

d. Tiles - 500 or so in 1 inch ceramic, wood, heavy cardboard. These are used in number concept development, counting, development of area understanding and an intuitive feel for multiplication and division.

e. Colored beans - 500 or so. These should be large lima beans, sprayed one color on one side and a different color on the other side. These are used in number concept development, counting, equality.

f. Microcomputers - If one that runs LOGO is available, work with LOGO can be used in connection with the

manipulative materials. Some selected problem solving software can also be used.

g. ASCOBLOCS - 2 sets. These are used for similarity and difference activities, classification and creative expression.

10. Gather storage containers for the materials. Ice cream pails, plastic gallon milk containers, paper cartons in 1/2 gallon, quart and pint sizes, plastic dishes with covers, different cans, bottles and boxes (such as cereal boxes).

11. Send letters to parents and arrange for a meeting with all parents if that seems desirable.

12. Familiarize yourself with procedures for conducting Piagetian task interviews. You will need to periodically assess children as to their progress in developing conserving behaviors relative to number, length, area, 1-1 correspondence, space relations, class inclusion and part-whole relations.

13. Read the Kindergarten Teacher's Guide to see what the children have had before. Many of these lessons can be repeated in the first grade, particularly early in the year.

## LEVEL ONE
## .RESOURCES OF USE

There are several materials that will be useful as resources to best use the Level One Teacher's Guide.

These are grouped by topic or activity.

Classification, Comparing and General Resources:

1. *Mathematics Their Way - M. Baratta Lorton*
   *Workjobs*
   *Workjobs II*
   *Addison Wesley Publishing Co.*

2. *Attribute Games and Problems - Marolda*
   *Creative Publications*

3. *Explorations 1 and Explorations 2*
   *Mathquest 1, Mathquest 2*
   *Addison Wesley of Canada*

Patterns and Pattern-Related Thinking:

1. *Pattern Block Problems for Primary People*

2. *Pattern Animals*

3. *The Pattern Factory*
   *Creative Publications*
4. *The Baratta Lorton* books and *Explorations*

Number Concept:

1. *Developing Number Concepts*
   *Using UNIFIX Cubes - Richardson*
   *Addison-Wesley*

2. *UNIFIX One Step at a Time - Number Concepts*

*Lakeshore Curriculum Materials*

3. *The Baratta Lorton* books and *Explorations*

Place Value:

1. *UNIFIX One Step at a Time - Operations and Place Value*
   *Lakeshore Curriculum Materials*

Problem Solving:

1. *TOPS Beginning Problem Solving K-1*
   *" Sam and the Storm at Willow Pond"*
   *"Fredericka and the Big, Bad, Biting Bee"*
   *"Maggie The Nervous Mouse"*
   *"When Barney Stopped Laughing"*
   *Dale Seymour Publications*

3. *Problem Solving Experience in Mathematics*
   *Grade 1 and Grade 2 - Charles Lester*
   *Addison Wesley Pub. Co.*

4. *Math Problem Solving Beginners through Grade 3*
   *Overholt, etc.*
   *Allyn and Bacon*

FIRST GRADE REPORT CARD

This reports on the progress of _____ toward achievement of the most important goals in mathematics learning in this grade.

NUMBER:
Has mastered the full meaning of the numbers up through _____
Understands the meaning of addition as an operation on numbers
Understands the meaning of subtraction as an operation on numbers
Can write a number sentence to indicate the meaning of an oral "story" problem

COUNTING:
Can orally count through _____
Can count the number of objects in a group of size up to _____
Can count on from a given number
Can count back from a given number
Can skip count by _____

PLACE VALUE:
Understands the difference between the column for ones and that for the group sized used
Can write two digit representations for groups and ones in a collection of objects
Understands "carrying" and "borrowing" as this involves forming a group when required and decomposing a group when required

PATTERNS:
Can copy a given pattern
Can add on to a given pattern
Can insert missing elements into a pattern
Can translate a pattern from one medium to another
Can identify a pattern within some configuration of things

CLASSIFYING:
Can sort objects with one property in common
Can sort objects with two properties in common
Can sort objects with more than two properties in common
Identifies how objects are alike and different

COMPARING:
Uses comparative language correctly as this involves:

space location length number size
area of similar shapes quantity height distance

Orders three objects according to (same as above)
Orders more than three objects according to (same as above)

FRACTION:
Knows the following fractional parts:

1/2 1/3 1/4 2/3 3/4 _____

**ASSESSMENT RECORD**
**for**

***Mathematician:*** _____

**Assessment:** "Please Count for me." (Knowing the counting words in correct sequence.)

**Circle the largest number in the sequence the child uses correctly.**

| Date: | Number Sequence |
|---|---|
| | 1 2 3 4 5 6 7 8 9 10 11 12<br>13 14 15 16 17 18 19 20<br>30 40 50 |
| | 1 2 3 4 5 6 7 8 9 10 11 12<br>13 14 15 16 17 18 19 20<br>30 40 50 |
| | 1 2 3 4 5 6 7 8 9 10 11 12<br>13 14 15 16 17 18 19 20<br>30 40 50 |

**Assessment:** "Please Count These." (rational counting of objects)

**Circle the** largest **number in the set the child correctly counts.**

| Date: | Number Sequence | Moves Objects | Touches Objects | Mentally Makes 1-1 Corres. |
|---|---|---|---|---|
| | 1 2 3 4 5 6 7 8 9<br>10 11 12 13 14 15 16<br>17 18 19 20 30 40 50 | | | |
| | 1 2 3 4 5 6 7 8 9<br>10 11 12 13 14 15 16<br>17 18 19 20 30 40 50 | | | |
| | 1 2 3 4 5 6 7 8 9<br>10 11 12 13 14 15 16<br>17 18 19 20 30 40 50 | | | |

**Assessment: Writing Numerals - (should be smooth and flowing)**

**Circle the numerals the child can correctly write:**

| Date: | Numerals |
|---|---|
| | 1 2 3 4 5 6 7 8 9 0 |
| | 1 2 3 4 5 6 7 8 9 0 |
| | 1 2 3 4 5 6 7 8 9 0 |

**Assessment: Counting backwards (with objects)**

**Place a number of objects in front of the child. Remove them one at a time. Each time the child should tell you how many remains. Circle the largest number the child can count back from.**

| Date: | Number |
|---|---|
| | 5 6 7 8 9 10 11 12 13 14 15<br>16 17 18 19 20 30 40 50 |
| | 5 6 7 8 9 10 11 12 13 14 15<br>16 17 18 19 20 30 40 50 |
| | 5 6 7 8 9 10 11 12 13 14 15<br>16 17 18 19 20 30 40 50 |

**Assessment:** Counting backwards (orally). Start with a number and ask the child to count back from that number to one or zero. Circle the largest number the child can successfully count back from orally.

| Date: | Number |
|---|---|
| | 5 6 7 8 9 10 11 12 13 14 15<br>16 17 18 19 20 30 40 50 |
| | 5 6 7 8 9 10 11 12 13 14 15<br>16 17 18 19 20 30 40 50 |
| | 5 6 7 8 9 10 11 12 13 14 15<br>16 17 18 19 20 30 40 50 |

**Assessment:** Number recognition (visual recognition of the number property of a collection. Give the child an array of dot cards with randomly arranged dots ranging from two to eight. "Show me ____."

| Date: | Number |
|---|---|
| | 2 3 4 5 6 7 8 |
| | 2 3 4 5 6 7 8 |
| | 2 3 4 5 6 7 8 |

**Assessment: Numeral recognition**

Show the child a numeral chart, point to numerals out of order on the chart, i.e. "7", first etc. Circle those numerals recognized.

Date: Numerals

0 1 2 3 4 5 6 7 8 9

0 1 2 3 4 5 6 7 8 9

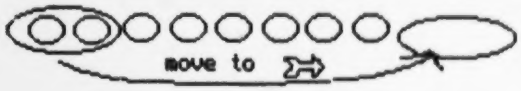
0 1 2 3 4 5 6 7 8 9

**Assessment: Conservation of one-to-one correspondence**

Arrange two rows of 5-9 objects. Have the child make sure there are as many in one row as in the other.

Rearrange one row to make it (1) longer, or (2) shorter, or (3) compact in area:

or (4) totally translated to the left or right, etc.




If the child is conserving, the response will always be "both rows still have just as many.", or something that means that.

| Date: | Conserves one-to-one correspondence | | |
|---|---|---|---|
| | 1-5 | 6-10 | 11-20 |
| | 1-5 | 6-10 | 11-20 |
| | 1-5 | 6-10 | 11-20 |

Assessment: Conservation of Invariance of number.

Place a group of objects before the child. Ask the child to count them. Rearrange the group by (1) putting into two or more distinct groups, (2) collecting into compact arrangement, (3) scattering so they are widely separated. In response to "how many are there now?" the child should indicate the number has not changed in some way.

| Date: | Conserves Invariance | | |
|---|---|---|---|
| | 1-5 | 6-10 | 11-20 |
| | 1-5 | 6-10 | 11-20 |
| | 1-5 | 6-10 | 11-20 |

Assessment: Estimation

Have four glass containers with 20-50, 100-125, 200-225 and 300-400 marbels or M&Ms or glass beads in them. Ask the child to estimate how many are in each. Record the responses:

| Date: | 20-50 | 100-125 | 200-225 | 300-400 |
|---|---|---|---|---|

Assessment: Part-part-whole in number. Show the child with beans or other objects equal to number being tested in an open hand. Ask the child to count these. Hid both hands behind your back and transfer some into the other hand. Show the child the original hand open again. Depending on how you have altered the number of objects, ask questions like: "How many are in my other hand?" "How many (two's) are in my other hand?", etc. Repeat until ALL part-part combinations have been tested for. Record the date on which a child indicates "knowing": each number.

Number 3 4 5 6 7 8 9 10

Date _____

---

Assessment: Symbolic level of numbers. Record the dates on which the child writes number sentences to show ALL of the part part combinations for each number, ie., 4 = 3 + 1 = 1 + 3 = 2 + 2 = 4 + 0 = 0 + 4 = 2 x 2

Number 3 4 5 6 7 8 9 10

Date _____

**Assessment:** Ordinality of number. Show the child a line of objects or picture of objects. (ducks in a row). Have the child point to the object described by an ordinal word such as "fith," "sixth," etc. Circle the highest ordinal word used correctly on the date.

| Date: | First Second Third Fourth Fifth Sixth Seventh Eighth Ninth Tenth |
|---|---|
| | First Second Third Fourth Fifth Sixth Seventh Eighth Ninth Tenth |
| | First Second Third Fourth Fifth Sixth Seventh Eighth Ninth Tenth |

## CLASSIFICATION

Materials to use are any that have recognizably different properties - color, shape, length, size, area, number of holes, textures, etc. If children have put together "Treasure Boxes", these can be used.

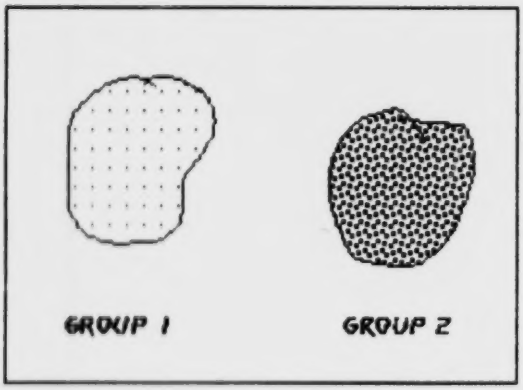
Generally start by sorting into TWO groups, observing which properties are chosen and whether that property is retained until sorting is complete. After much experience sorting into TWO groups using a variety of properties, the sorting should be done into THREE groups.

Questions to ask:

How are the objects in each group alike?
How are the objects in each group different?
What are the differences between groups?

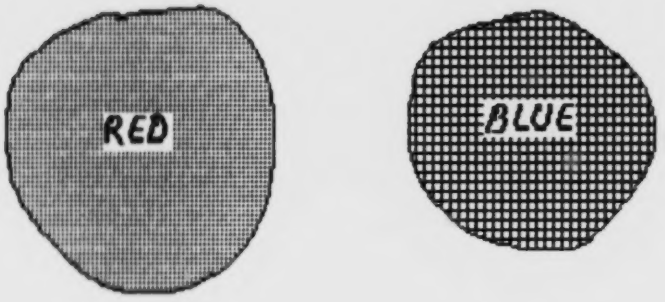
A major goal is to eventually get them to recognize MULTIPLE CLASSIFICATION - the joint presence of two or more properties - red AND square.

During later stages, have children draw the objects put into each group using forms such as:




Spend a lot of time talking about, with the children - SIMILARITIES (ALIKE), DIFFERENCES, AND

Children should also be able to use the either-or meaning of OR




The objects within the boundaries are EITHER red OR blue - none is red AND blue.

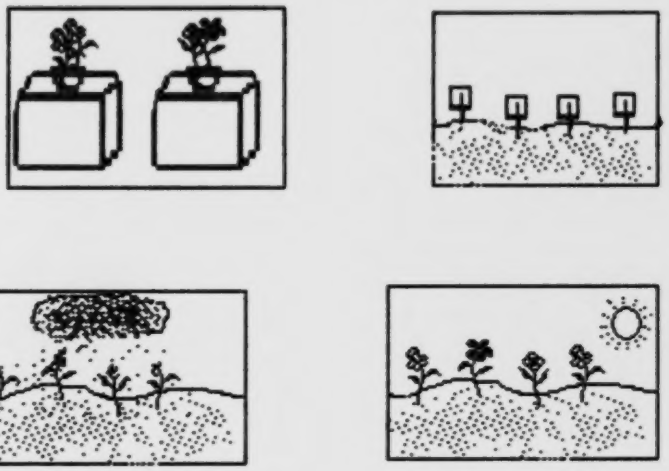
Children should classify a collection of objects once every two weeks or so at LEVEL ONE and early in LEVEL TWO.

LEVEL ONE

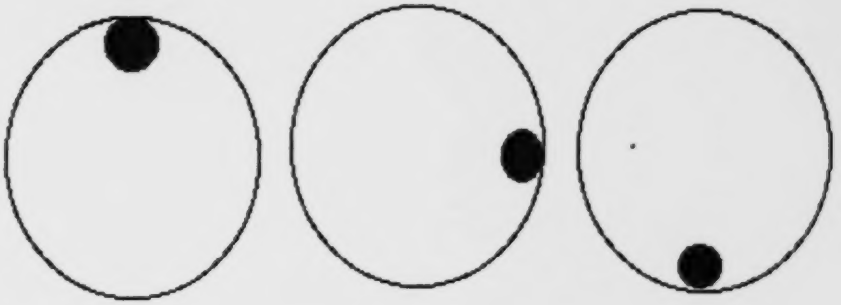
SAMPLE PROBLEMS:

Classifying: You want to make a tree house. Which one would not be needed:

Ordering: Put the pictures in order.



Draw what comes next:



| A matching test: | Problem type | Two number sentences |
|---|---|---|
| Example: | Fred has 9 crayons and Tom has 6 crayons. Fred has how many MORE crayons than Tom? | 9 - 6 =<br>9 + 6 = |

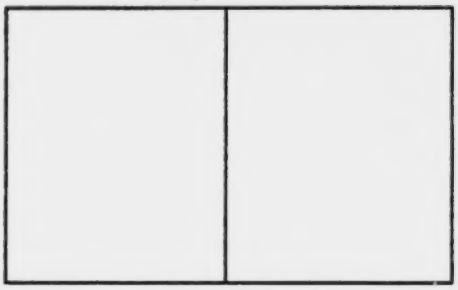
LEVEL ONE

SORTING AND CLASSIFYING: BEGINNING

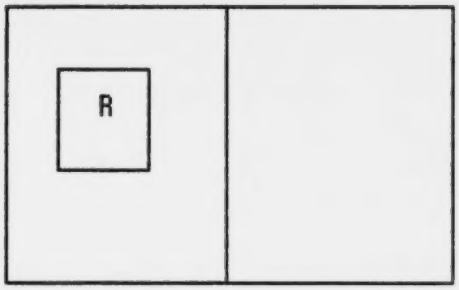
Materials to Use: Materials with shapes, colors and of different sizes. Concrete materials that work well are ASCOBLOCS. Overhead transparency materials can be made that have shape, color and size.

Introduction: With ASCOBLOCS and smaller groups of children:

Make a split mat:

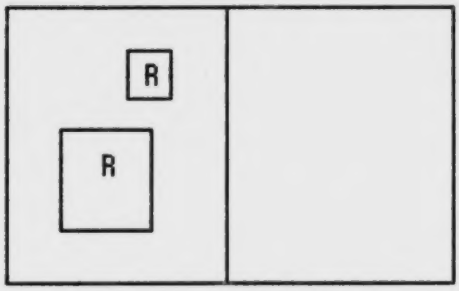


Place this on the floor. With the children seated around you, place a large red square on one side:

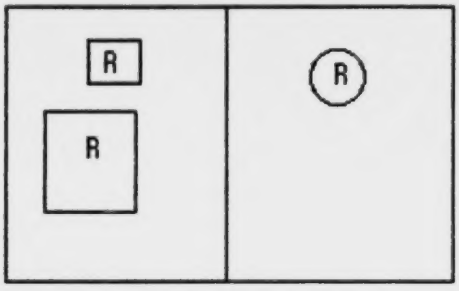



Pick up a small red square and ask the children if it goes with the one there, or on the other side.

If the children decide collectively it goes with the first, put it there. If the children express different opinions, probe to find the reasons for their choices. What properties are they considering? Is color or shape predominant in their thinking? Do they consider size?




Now pick up a small red circle and ask the children where it goes. If they are consistent in sorting by shape, they should tell you to place it on the other side. If any other responses are given, probe for the reasons for these.




Continue with other pieces, always asking questions to find reasons for the decisions.

With overhead transparency pieces and a large group of children:

Put a split mat transparency on the overhead projector. Follow the same procedure as above with the concrete materials.

Keep reminding the children that once they begin sorting using a particular attribute or property, they must use this until sorting is complete.

Extensions: Repeat this occasionally using a different piece to start and selecting pieces next to make the children consider the properties of shape, color and size.

SORTING AND CLASSIFYING: More Common Properties

MATERIALS TO USE:

1. The students themselves and what they are wearing

2. Treasure Chests - individual, and class gathered materials for the group to use.

INTRODUCTION:

1. Have the students sort themselves by going into the same group if they are wearing a blue shirt or blouse. Use other characteristics for such sorting like:
   a. wearing glasses
   b. wearing sneakers
   c. button down fronts on tops
   d. collars
   e. white stockings
   f. a ring, etc.

The emphasis is on HAVING a property and NOT HAVING that property.

Have the students suggest properties to use.

TREASURE CHESTS: Each child is to sort the contents of his Treasure Chest. Choose the property initially to use to sort into two groups. Some examples are:

metal and non-metal
one-hole and other
plastic and non-plastic
round and non-round, etc.

Then give the children the chance to choose the sorting property.

EXTENSIONS:

1. When children can sort easily into two groups, have them sort into three groups.

2. Do "silent sorting". Put a UNIFIX link upright on a table. The children are to remain silent as long as it is upright. Sort a set of shapes on the

overhead or flannelboard into two groups. Tip over the link and ask the children what property was used to sort them.

3. "Read my Mind", p. 70 of Mathematics Their Way.

4. Property Stroll. Decide on a property they are to look for - round, square, red, etc. Take the children for a stroll through the school after informing the children which property they are to look for. Upon return, ask the children to draw and color everything they saw with that property. Use other than rigid physical properties like things that move, signs with words, new things, things with 4 of something, etc.

5. Some properties useful for sorting (and subsequent graphing!):

favorite foods
favorite drinks
favorite pets
favorite colors
favorite songs

6. Other things to sort:

pictures cut out of magazines

| | |
|---|---|
| shells | buttons |
| seeds | bottle caps |
| beans | plastic toy cars, animals, etc. |
| leaves | nuts |

LEVEL ONE

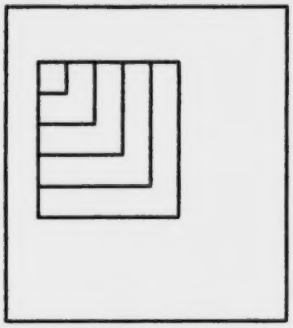
## PATTERNS

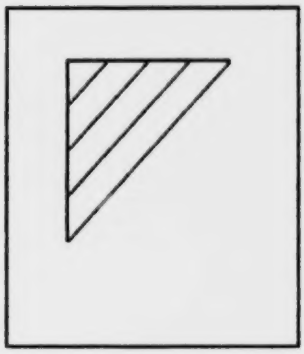
Background: Children should have had experience in Kindergarten in creating, copying and extending patterns (see the Kindergarten Level Lesson Plans.)

Repeat some of those activities. The emphasis at this level should be on translating patterns, using patterns for problem solving, inserting missing elements into patterns and describing patterns.

Activities:

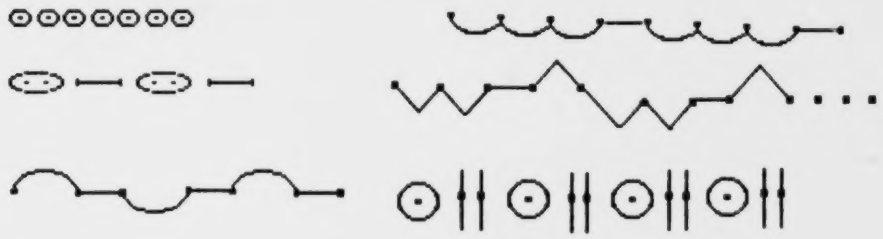
1. Have the children make different people patterns - sit,stand - arms out, arms in - arms up, arms down - stand, kneel - stand on left leg, stand on right leg - left arm up, right arm up, etc.
2. Use sound to material patterns such as "snap, snap, clap" with the children arranging materials - toothpick, toothpick, bottlecap, etc.
3. Use letter to material patterns such as ABBABB to red, blue, red...UNIFIX cubes.
4. Use sound to numeral or letter patterns such as snap, clap, clap, stamp to ABBC or 1223.
5. Have children build "roads" from Pattern Blocks or Attribute Blocks.
6. In response to oral or written stimuli, have children make patterns of:
   a. Pattern Blocks
   b. Treasure Chest materials
   c. Colored beads
   d. Paper shapes
   e. Constructionn paper chains
7. Have children collect a variety of patterns - fabric scraps, wallpaper samples, Christmas stickers, on dishes, on silverware, on shoe soles.
8. Initiate a pattern on the overhead with some materials. Have the children extend it with the same materials.
9. Have the children make patterns on the Geoboard. Some samples are:



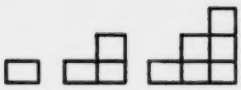


Have them count the squares added with each new, larger shape.

10. Have the children make dot line patterns, i.e.:



11. Have the children find patterns in tables:



Building Stairs of Cubes

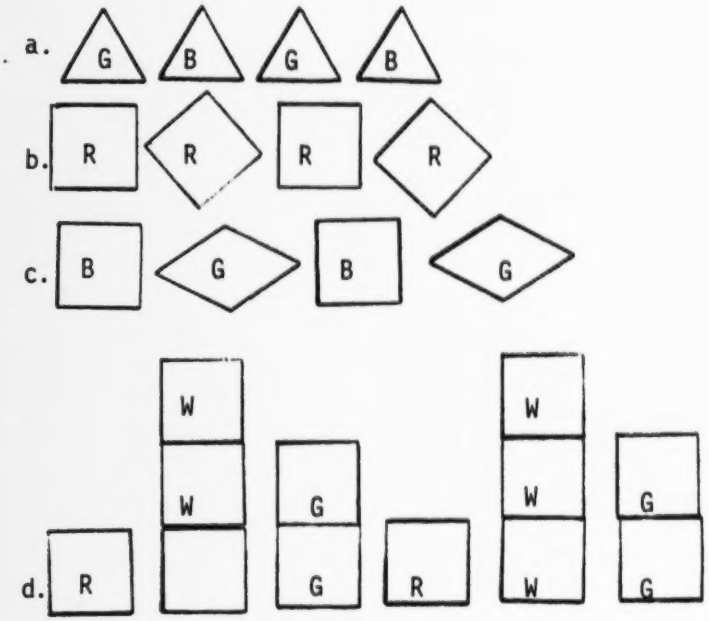
How many cubes to build 4 stairs?
5 stairs?

| stairs | cubes |
|---|---|
| 1 | 1 |
| 2 | 3 |
| 3 | 6 |
| | |

12. Have children make patterns with materials so that these have:
    a. change of color as the pattern

b. change of spatial position as the pattern
c. change of position and color as the pattern
d. change of number and color as pattern

Examples:




13. Given UNIFIX links of different colored patterns, pick out those with the same pattern.

LEVEL ONE

## NUMBERS

Introduction: Children find numbers everywhere in their environment. Here are some questions to pose for children to think about and investigate:

Why do houses have numbers on them?
Why do cars have license plates with numbers?
Why are streets numbered?
Why are classrooms numbered?
How are numbers used in stores?

Have the children keep a notebook or file folder for each number. Into it they should put all pictures or recordings involving that number that they find at home in magazines or record observations of as they ride in the family car, etc.

LEVEL ONE

WRITING NUMERALS

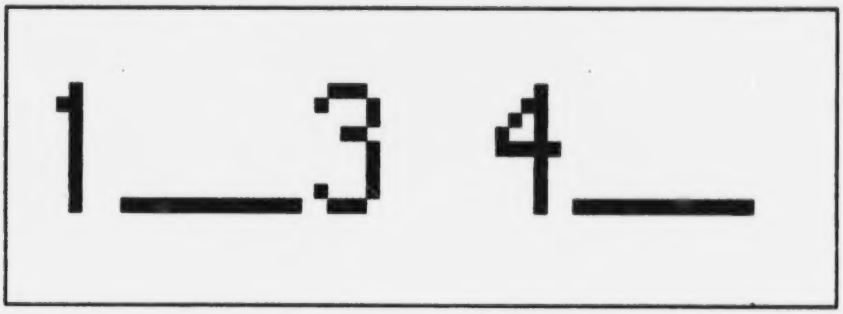
Background: Since writing what you know about numbers, number sentences and number operations must precede trying to decode someone else's use of symbols, knowing HOW TO WRITE numerals and the symbols for equality and the arithmetic operations, as well as WHAT THESE MEAN, is important to the work of first grade.

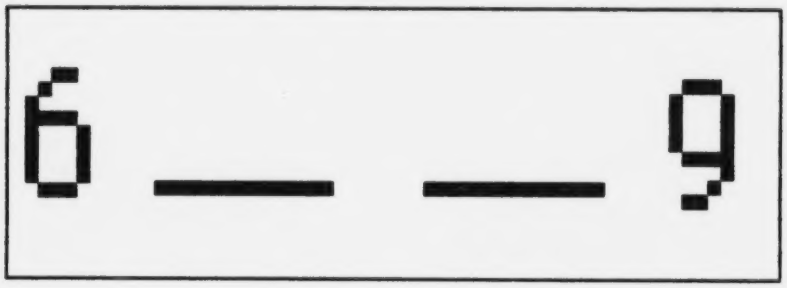
If your assessment of incoming performances indicates work is needed in this area, you will have to provide it.

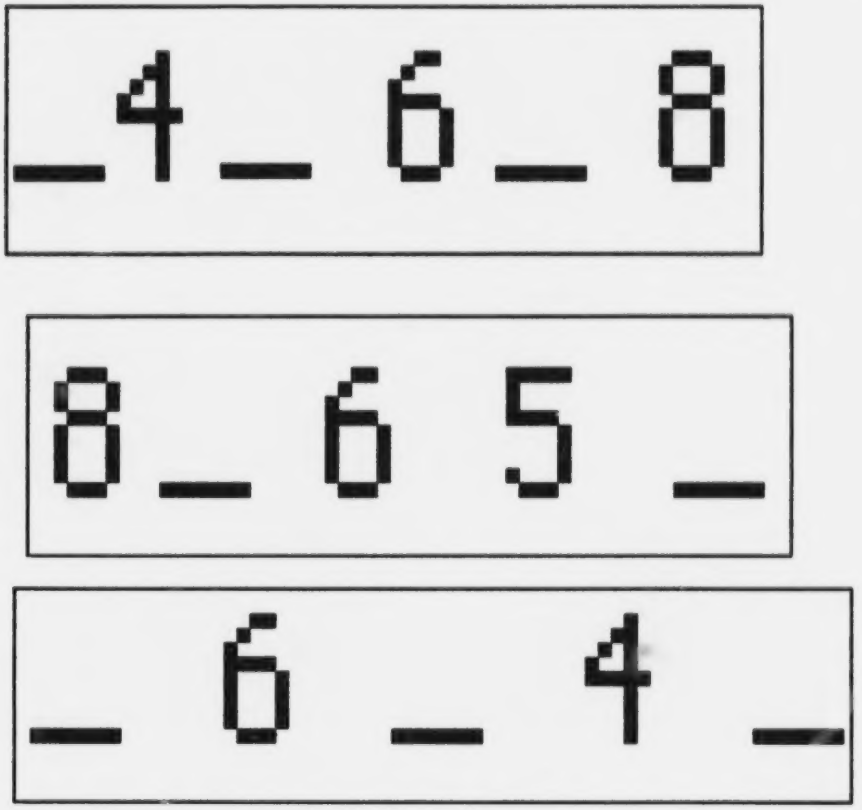
1. See the Kindergarten lessons on writing numerals (attached).

2. Other resources for ideas on writing numerals include Mathematics Their Way, pg. 43 through 51.

3. Make cards to combine numeral reading and writing like those shown and cover with acetate. Larger cards like these can be used for small group oral work.






4. Make overhead transparency versions of these cards and have the children write (on lap chalkboards) the missing numerals.

KINDERGARTEN

NUMBER CONCEPT 9 : WRITING NUMERALS

Background: Children should learn to write all numerals during kindergarten. This is best done by first writing numeral with large muscles and large movements and gradually moving to hand and finger muscles and movements.

LESSON: On large (2' x 3' or so) pieces of tagboard, railroad board or heavy wrapping paper, print the numerals **0,1....9,** so they are very large. Hang one numeral board at the front of the room. Have the children stand at a distance from the chart and ***trace*** the numeral with hand and arm. Do this for all numerals.

Follow-Up: Individually have children gradually move closer to the chart while tracing the numeral. When they get next to it they can trace the numeral with the forefinger.

When all children have ***internalized*** this large motor tracing, move to having them use fingers, crayons, large pencils to trace outlines of the numerals, dot patterns of the numerals, etc. Some suggestions are:

1. Trace numerals in soft clay

2. Trace numerals in wet sand

3. Trace numerals in dough and bake

4. Trace numerals made of sandpaper

5. Complete dot patterns of numerals (see Mathematics Their Way black line masters)

6. Practice writing numerals on lap chalkboards.

7. Have children "write" numerals on each other's backs with their forefingers. The one who is the "blackboard" must identify the numeral.

8. Trace numerals cut out of heavy cardboard.

Trace the opening, others trace the numeral cut out:

LEVEL ONE AND LEVEL TWO

USE OF SYMBOLIC FORMS

Children cannot use symbols or attach meaning to symbols without

(1) Development of the related concept through extensive experience at the concrete level;

(2) Careful attachment of symbols to the concepts; and

(3) Generation of symbolic forms to show understood concepts

prior to interpretation of symbolic forms. The relative amounts of time to be spent at each stage of this learning of the use of symbols is shown below:

| Concept Development at the Concrete | Connecting Symbols to Concepts | Generating Symbolic Forms | Interpreting Symbolic forms in in materials | Responding to Symbolic form with symbols |
|---|---|---|---|---|

Technically: numerals and +, - , =, etc. are "signs." They become symbols when the user has attached these to concepts, so that they symbolize for him some real situation.

LEVEL ONE

NUMBER ASSESSMENTS:

1. Children must know the correct order of number names in sequence (Rote Counting).

2. Children must learn to associate the number words with collections of objects by touching one object at a time as they say the words. The words and objects MUST BE in one-to-one correspondence.

3. Children must be able to easily write the numerals from 0-9.

4. Children must have enough experience with objects so that they can "see" numerosity up to fiveness.

5. Children must realize that the number property of a collection is invariant under rearrangement - that only adding something to it or taking something away from it will change the cardinality.

6. Children must be able to rationally count backwards by giving the name of the number left as objects are removed one at a time from a collection.

7. Children must identify all numerals from 0 to 9 correctly with the word name.

8. Children must associate the numerals 0-9 with numbers of objects of these sizes by pairing the numerals with object collections having those numbers.

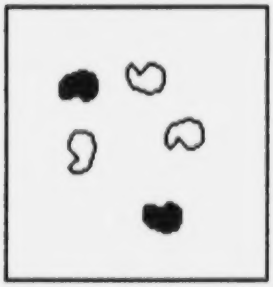
9. Children must internalize numbers to the extent that they can identify all of the part-part-whole combinations for each number one to ten (initially).

10. Assessment of ALL aspects of number must take place continuously with each individual child when time is available throughout the year.

LEVEL ONE

INTRODUCING NUMBER STATIONS:

BEANS: (2 PART COMBINATIONS OF NUMBER):
Show the children the colored beans. Roll out five and record the results on an overhead transparency version of the recording form.

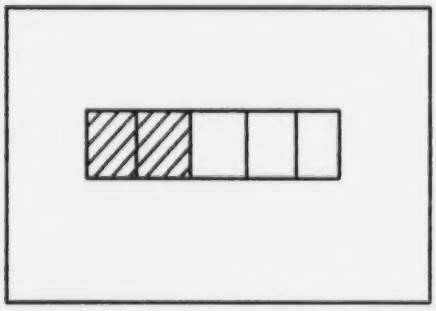


This is a two part combination

2 + 3

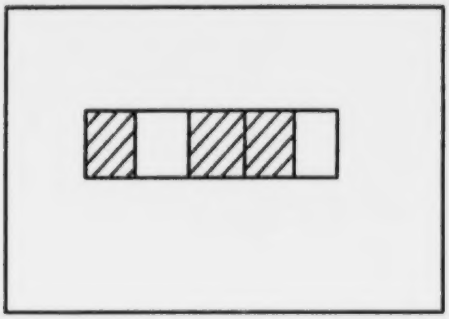
Tell them and demonstrate how they will be tested for each number when they have completed all of the number stations. (The hand game; lift the bowl; p. 186-87 Mathematics Their Way).

UNIFIX CUBES (PART-PART COMBINATIONS OF NUMBER):
Show the children a link of five Unifix of two colors. Color in one link on a transparency recording form.
Make a second link and show it. Color in a second section of the transparency recording form.



This is a two part combination 2 + 3

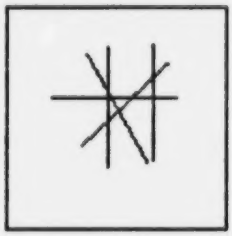
Make a second link and show it. Color in a second section of the transparency recording form.



Note this is a 4 part combination

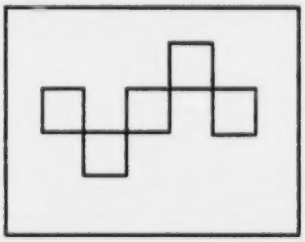
(1 + 1 + 2 + 1)

TOOTHPICKS:
Show the children five toothpicks. Arrange these in any random fashion on the overhead projector.



Explain that recording is done directly by pasting the five toothpicks on a sheet of paper. Another five toothpicks can be arranged in a DIFFERENT way and pasted and so on.

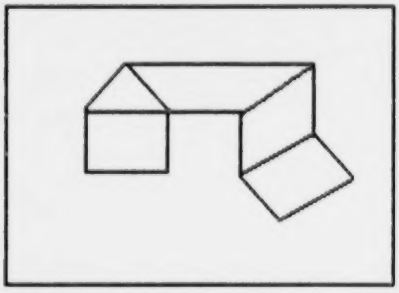
SQUARE TILES (CERAMIC, WOOD, POSTERBOARD):
Place 5 square tiles in some configuration on the overhead projector.



Place a small piece of overhead graph paper transparency on and color in squares to correspond to your pattern of squares. Rearrange the squares and

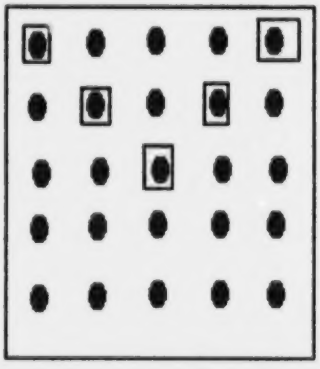
color in this arrangement on another small piece of graph paper. Point out that this is what will be done at that number station.

PATTERN BLOCKS:
Use of overhead transparency versions of Pattern Blocks and arrange five of them in a pattern.



Show the construction paper pattern blocks to the children and explain how to paste these on a large piece of construction paper to make a record of the pattern created.

GEOBOARDS:
Show the children a large (12" x 12") Geoboard. Place five paper squares on pegs in some arrangement.



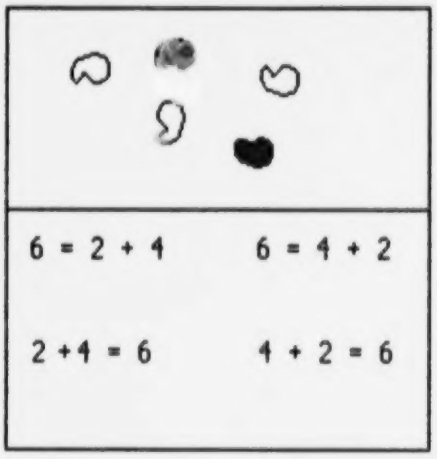
Put a dot paper transparency on the overhead projector and circle dots in the same rows and columns as the squares on the Geoboard. Tell the children this is how the arrangements of paper squares on the Geoboards are to be recorded at that number station.

OTHER MATERIAL:
For each other material that you introduce at number stations - contents of "treasure chests", colored beads, wooden cubes, etc., first show the children

the materials, how they are to be used and how this is to be recorded (if that is to be done).

NUMBER STATION ACTIVITY:
While children are working at number stations, walk around and observe this. Frequently ask leading and probing questions to encourage exploratory activity by the children.

NUMBER STATION FOLLOW UP:
When children have learned how to write numerals and combinations of numerals and signs in number sentences, add this to the recording of part-part-whole relationships such as when using Unifix and colored beans.



LEVEL ONE

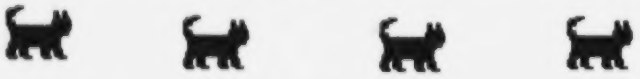
NUMBER CONCEPT

Background: A child has a fully developed concept of number when all of the components have been integrated, the relationships between them recognized, and number is used appropriately. Number has a part-part whole component, an invariance under physical rearrangement component, an ordinal component, a seriational or cuccessor component, an inclusion component, and a 1-1 correspondence component.

Part-Part Whole: All numbers can be made of parts consisting of smaller numbers in many ways. This 5 = 2 + 3, 4 + 1, 2 +2 + 1, 1+ 1 +1 + 1 + 1, 3 + 1 + 1, 2(2) + 1, 1 + 2 + 1 + 1, etc. Children must master all of these part-part whole relationships for EACH number en route to developing number concept.

Invariance: The cardinality, or number property, of a collection remains invariant under physical rearrangement of the elements. The cardinality does not change unless some element is ADDED TO or REMOVED FROM the collection. Use of materials that indicate a particular arrangement of countable things such as dominoes or UNIFIX trays really inhibit the development of this recognition. Materials that can be arranged in a variety of ways such as toothpicks, buttons, bottle caps, tiles, beans, cubes, etc. help in this recognition.

Ordinal: The ordinal property of number is the use of number to describe location in a sequence, series or order. Examples are "first", "eight", etc.:



Third

Seriation or Successor Relationship: This is the property of our counting number system wherein each successive number is one more than that before it. This is a property that doesn't exist in "dense" systems such as fractions where you can always find a number between any two. An example is 7/8 and 15/16. The mean, or average, of these is found by dividing their sum by two which yields a number between: 7/8 + 15/16 = 29/16 : 2 = 29/32.

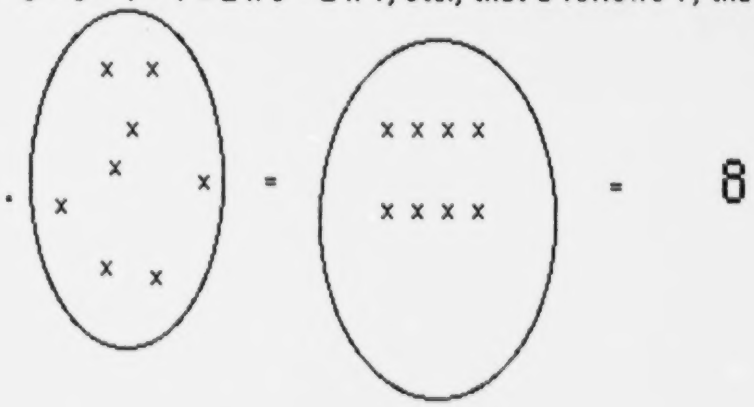
The counting numbers are successive. Two follows one, and there is NO counting number between these.

Inclusion: Each counting number is "included in" its successors in that it is a part of each successor. 3 is included in 4 since 4 = 3 + 1; 3 is included in 5 since 5 = 3 + 1 + 1 or 3 + 2. When numbers are larger enough a smaller number may be a repeated part of it. There are 2 (2)'s in 4 since 4 = 2 + 2.

One-To-One Correspondence: Identification of a one-to-one correspondence between the elements of TWO collections is fundamental to (1) recognizing that two different collections have the same number property, and (2) establishment of equality between two quantities. The former is important for a child's recognition that number is an ABSTRACTION independent of the kind of material or location. The "fourness" of the legs of a chair is the same "fourness" as the "corners" of a square.

Because these components come together slowly in a child's thinking, it is necessary that they have repeated experiences with these components, one number at a time. That is the purpose of the NUMBER STATIONS.

It is equally important that each child be periodically checked for these components, that is the purpose of the "hand game" checks, etc.
A child knows "eight" when it is clear that 8 is made of parts in many ways:
8 = 5 + 3 = 6 + 2 = 7 + 1 = 2 x 4 = 8 x 1 = 5 + 2 + 1 =
3 + 3 + 1 + 1 = 2 x 3 + 2 x 1, etc., that 8 follows 7, that:




and the eighth position is after the seventh position. Much repetition is needed for this to develop.

LEVEL ONE

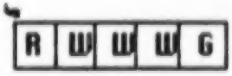
NUMBER CONCEPT 2: Extended Numbers of Parts

For numbers of five and larger, children should look at 3 and 4 part combinations also. This can be done using 3 and 4 colors of UNIFIX cubes.

Put 3 colors of squares on the overhead projector. "Let's see how many different ways we can make FIVE out of these three colors of squares. One way is:

Write 5 = 2 + 1 + 2 + 1

"Can you see another way?" Either:

or 

is likely to be suggested. Write: 5 = 3 + 1 + 1

"Are there any other ways to use different numerals to show five?" "We could put them in a different order, but would still have five:

5 = 2 + 2 + 1 = 2 + 1 + 2 = 1 + 2 + 2

and

5 = 3 + 1 + 1 = 1 + 3 + 1 = 1 + 1 + 3

"There are two ways to make five of three parts." "How many ways can six be made of three parts?"

Write down ALL possibilities the children indicate, including different orders of the same digits. The different ways are:

6 = 3 + 2 + 1 = 2 + 2 + 2 = 4 + 1 + 1

a total of three ways - one more than for five.

Pass out the recording forms and UNIFIX cubes of three different colors to each pair of students. Circulate and see how the number sentences look. Point out where the same three digits are ordered in a different way.

Extension: Use four colors of cubes for numbers six and larger so children can have experience with 4 part combinations to make numbers. An example is: 6 = 3 + 1 + 1 + 1.

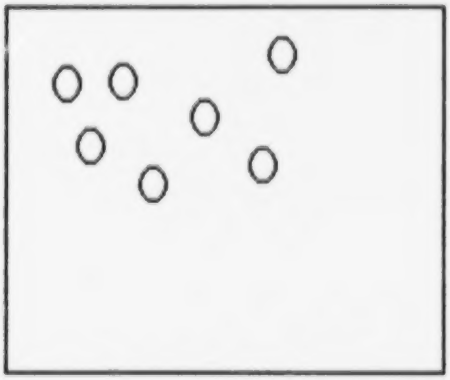
LEVEL ONE

NUMBER CONCEPT ACTIVITIES

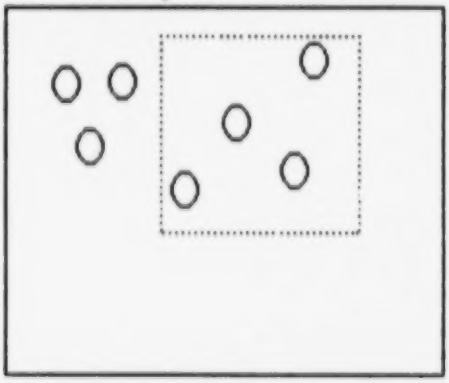
Background: In addition to developing the various components of number into a complete concept through students working individually at number stations, you should have group number concept development activities.

LESSON ONE - Hidden Numbers:

Place a number of chips on the overhead projector or on a flannel board. Use a number that is one many children are working on, such as seven:



Give the children an opportunity to identify the number. Ask how many chips are on the projector. Then place a card over some of them:



Ask the children how many are under the card. Do this periodically with different numbers, and by covering several different subsets for each number.

LESSON TWO - "Change Up"

The children should have work mats and some countable materials - buttons, inch cubes, UNIFIX cubes, etc.

For a selected number - like six - have the children place that many objects on the mat. Then ask them to change it in specific ways:

"Make two groups of three"
"Make three groups of two"
"Make four and two"
"Make a five and one"
"Make six one's"

For odd numbers, there will be remainders in some cases. Discuss these. "How many more would you need for another _____?", for example.

When using seven, you might ask the children to do the following:

"Make as many two's as you can"
"Make as many three's as you can"
"Make one four and whatever is left - what is left?"
"Make one five - what is left?"

LESSON THREE:

Materials - Each child should have several individual UNIFIX cubes, not all the same number, of four different colors. These can be in a box or paper sack.

Activity: Have each child dump out the contents of the box or sack on the floor or on the desk.

"Count the Unifix Cubes" - solicite from each child the number of UNIFIX cubes. Ask the children if they remember who has the highest number; the lowest number. Ask if any two or more have the same number of cubes.

"Sort your UNIFIX cubes by number."

"Link together those of the same color."

"Put your UNIFIX links in order from the shortest to longest."

"Do any of you have EQUAL links?"

"Join together the two shortest links."

"How many UNIFIX are in the new link?"

"Which part of the new link has the most cubes?"

"Join this link to the next longer link."

"How many parts does this link have?"

"Which part is the smallest?"

"How many UNIFIX cubes are in the largest part?"

"Link together ALL of the UNIFIX cubes." "How many cubes are in this?"

"Break off as many TWO links as you can from this."

"How many TWO links did you get?"

"Arrange the TWO links in rows.": (Example)

etc.

"Link all of the UNIFIX together again."

"Break off as many THREE links as you can."

"How many THREE links did you get?"

"Did you get more, as many as, or fewer THREE links as TWO links?"

"Put the THREE links in rows?"

"Link all of the UNIFIX together."

"If you made FOUR links, would you get more than or fewer than the THREE links?"

"Link your UNIFIX together in links of the same color again."

"How many more cubes are in the longest link than in the shortest link?"

"How many links would you have to add to the shorter link so it is as long as the longest link?"

"Make a TWO link."

"Make a link THREE times as long."

"How many cubes are in the new link that you made?"

"How many TWO links can you get from this new one?"

"Make as many TWO links as you can."

"Join two of these together. How many cubes do you have?"

"Join another link to this longer link."

"Now, how many cubes do you have?"

"Is this link THREE times the TWO link?"

"Make a link FIVE times the TWO links."

(Repeat this sequence with THREE, FOUR, etc. links)

Extensions: Do this frequently with varying numbers of cubes of each color for each child.

Have them break off link sizes for numbers as large as they can handle.

Have them make links 2, 3, 4, 5, etc. times as long as given links.

Have them count the results frequently.

LEVEL ONE

## CALENDAR MATH

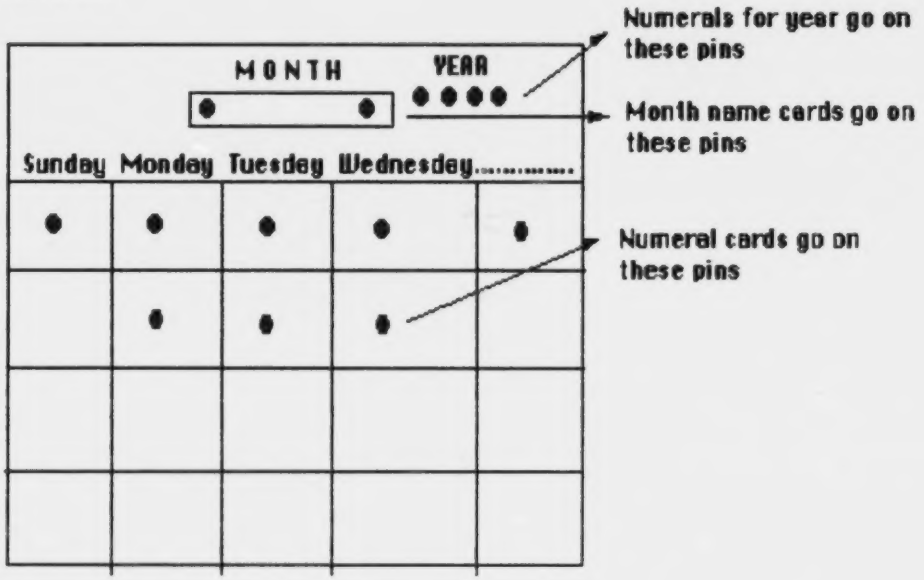
On . bulletin board place the following items. The diagram indicates a possible arrangement.

- -a number line with numerals for numbers 1 through 31
- -a calendar on light plywood or hardboard or heavy posterboard with nails as shown
- -a tens-ones chart
- -a tens frame chart for unifix cubes made from the same material as the calendar and with nails in the center of each square of the ten frame

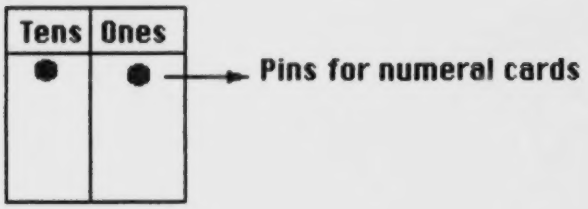
Number Line

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 .....31

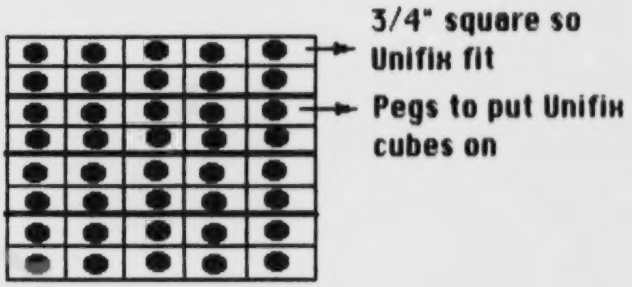
Calendar


MONTH
YEAR
Sunday Monday Tuesday Wednesday.....
Numerals for year go on these pins
Month name cards go on these pins
Numeral cards go on these pins

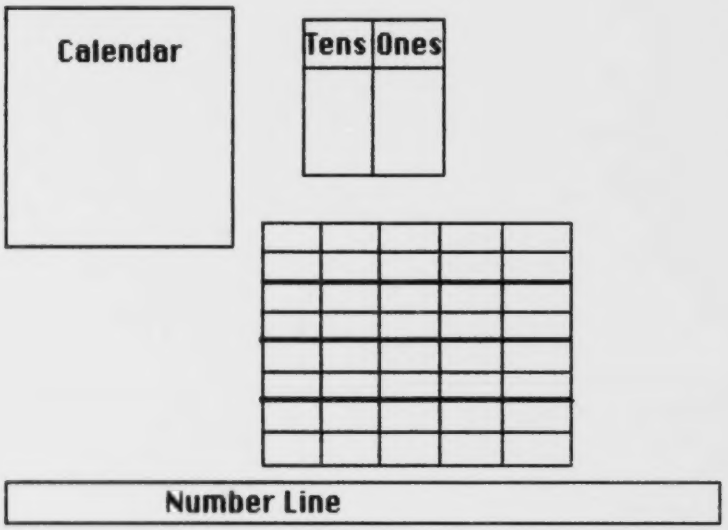
Place Value Chart




Ten Frame Chart




A Possible Arrangement



How to use the calendar arrangement. Ask students to either tell you where to put the materials or put the materials:

"What is this month?"

"What day of the week is this?"

"Which day of the month is this?"

Hang a date card on the number line

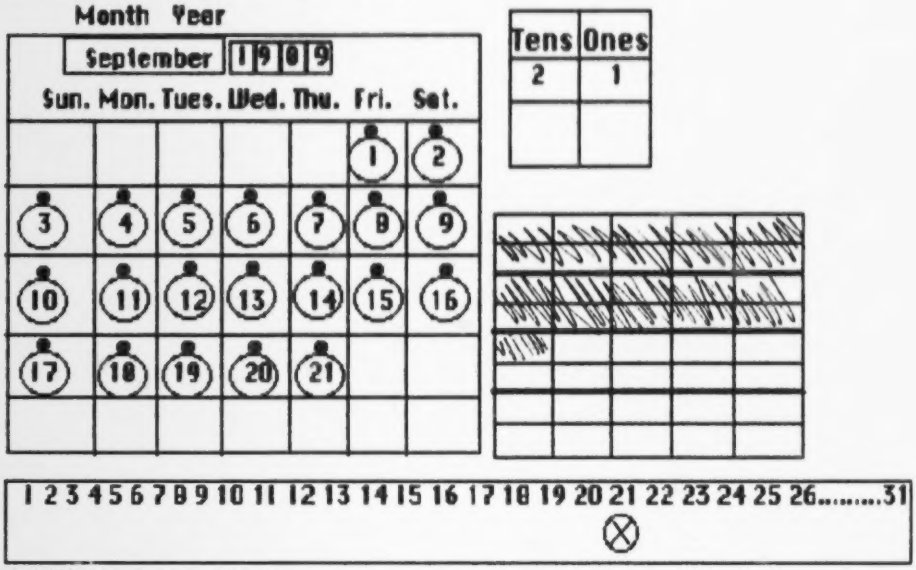
Hang a date card on the calendar

"What numerals go into the Tens-Ones chart?"

"How many unifix cubes go into the Ten-frame chart?"

"Write number sentences that equal the number of this date. Write as many different number sentences as you can."

Example: September 21, 1989




Additional questions:

"How many weeks of _____are gone?"

"How many _____(days of week) have we had so far in_____?"

"How many days are left in _____?"

"If _____has _____day, what day of the week will the last day of the month be?"

"On what dates are the _____in _____?"

Do this activity at the beginning of each math period before the counting and regular lesson. Coordinate your graphs - weather, birthday, etc. with the calendar activities.

## USING TEN FRAMES

Background: Ten frames enable you to relate numbers to 5 and 10. For example, to think of 7 as 5 + 2 or 10 - 3. When base ten is emphasized and children use base ten blocks, they will use place value boards that have ten frames on them.

### LESSON ONE: Combining, Comparing Ten Frames

Introduction: Use an overhead projector transparency of ten frames made from the master provided.

"What number is in the frame in C? E? F? etc."

"How much is A added to C? E added to F? etc." (There are over 35 such combinations to use.)

"What is the difference between B and C? G and E? etc."

This lesson should be repeated often during the time number concept is being developed.

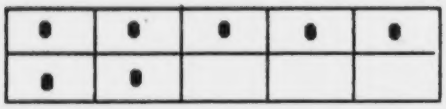
### LESSON TWO

A template is provided to make ten frames from. This master can be duplicated onto material similar to the master so students can use them. Each child should have two sets of frames from zero to TEN. Ten frames in the form of bean sticks can also be used. Beansticks are made by gluing beans onto tongue depressors.

The Game of War: Children are paired. Each has a set of ten frames. They choose one ten frame each; lay this upside down in front of them. Then turn these over. The one with the largest number wins. This can also be played with numeral cards. This gives children a chance to judge the numbers sizes from the symbolic forms.

### LESSON THREE

Introduction: Use a single transparency ten frame on the overhead, e.g.:



"How do you make ten from this?" (add three)

"How do you make five from this?" (subtract two)

Substitute different numbers into: "How do you make _____" to give mental practice in adding and subtracting.

LESSON THREE:

Each child should have a blank ten frame and UNIFIX cubes.

"Add two." The child puts two UNIFIX cubes on the ten frame.

"Add four." The child puts four more UNIFIX cubes on the ten frame.

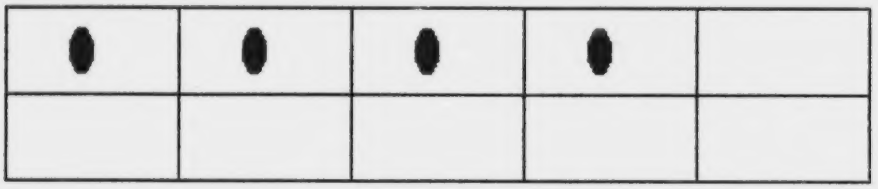
"Subtract 3." This child takes three UNIFIX cubes off the tne frame.

This count off - count on activity can be done early in the year along with other number concept activities.

A

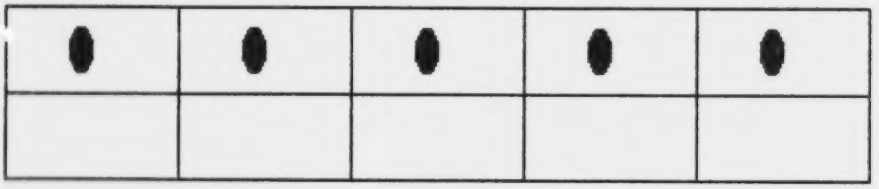
B

C

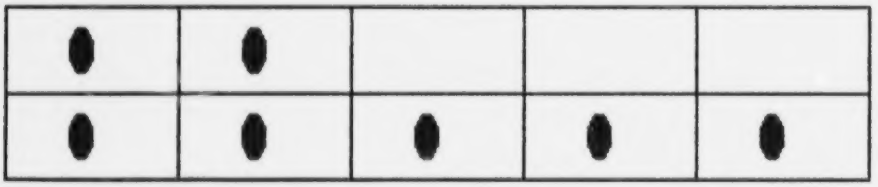


D

E



F

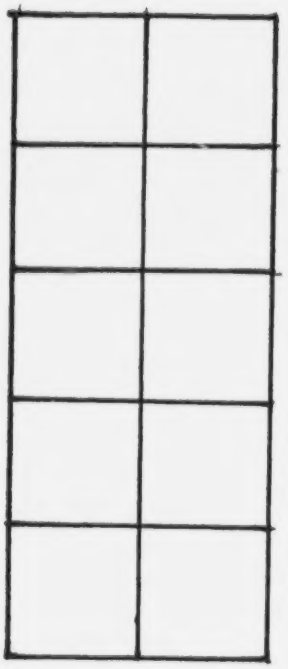


G

H

I

A TEN FRAME TO HOLD UNIFIX CUBES



LESSON FOUR - Measuring Things

Have the children use plastic links, large paper clips or UNIFIX cubes to measure short objects in the room - lengths of books, desks, etc. Gradually choose lengths that require more measuring units as the year progresses. Then have them use the chains or links in the following ways:

Count the units. Compare the same lengths measured by two different units and discuss these. Find how many two's, three's, etc. are in the chain or UNIFIX link. Orally emphasize cases like 8 two's are in 16; 5 threes are in 15, etc. 16 divided into twoooo gives 8 two's, etc.

LESSON FIVE

Have the children draw a numeral on a large piece of paper. Now have them use this as the basis for a picture.

LESSON SIX - Unifix

Show the children prepared links of UNIFIX cubes, e.g. one "five" and one "three". Ask them to tell you how many UNIFIX cubes there are. Use different links on different occasions. As the year progresses, use three links.

COUNTING: Counting Words

Background: How you approach oral counting experiences, will depend upon the results of your assessments of the counting skills of children. The suggested order of counting activities is given below. Children should have experience counting orally every day. Count out things distributed in class, children placed into groups, etc.

1. Counting up from one as far as possible. Gradually introduce each new number. Be especially careful when introducing the teens and decade numbers. Children seem to have particular difficulty with the interval "eleven....thirteen" and at each decade.

2. Counting on from given numbers. Many children will need a lot of help with this. Some will still orally "go through" the sequence of words leading to the selected beginning number. They must learn to subvocalize these, then finally eliminate this step and count from that number.

3. Counting by two's, three's, etc. This requires children to subvocalize those not to be orally given. It also gives experience that later enables them to internalizee multiplication "facts." This is essential for understanding of place value and mental computation later.

4. Counting back from starting numbers smaller than ten and gradually moving up to larger numbers in the counting sequence.

5. Counting back from given numbers in intervals, i.e., twelve, ten, eight, six, etc.

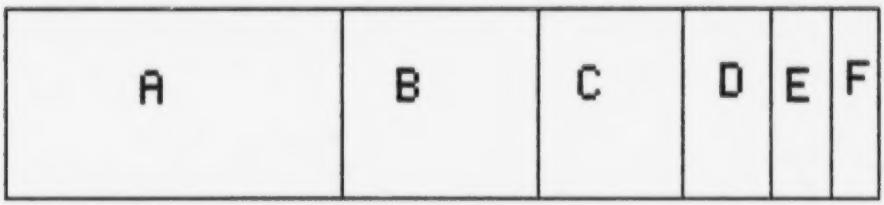
6. What comes before? "Twenty-six", etc.

7. What comes after? "Thirteen", etc.

8. Have children count off for teams in physical education, etc.

SYMBOLS FOR NUMBER

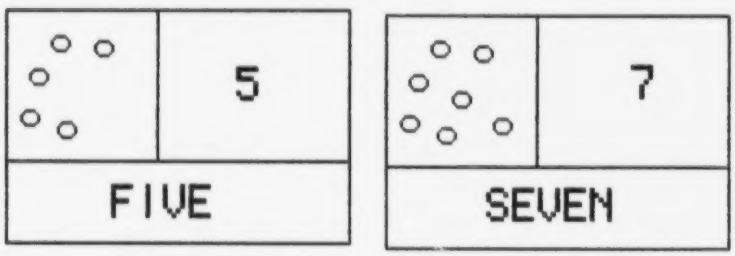
Background: Symbolic functioning in children develops in stages. It has become increasingly clear in areas where symbols are important - mathematics, reading, writing - that premature emphasis on symbol interpretation actually inhibits later understanding. Consequently, in this program, the recommended allocation of time for the connecting of symbols to concepts and the use of symbolic forms is as follows:



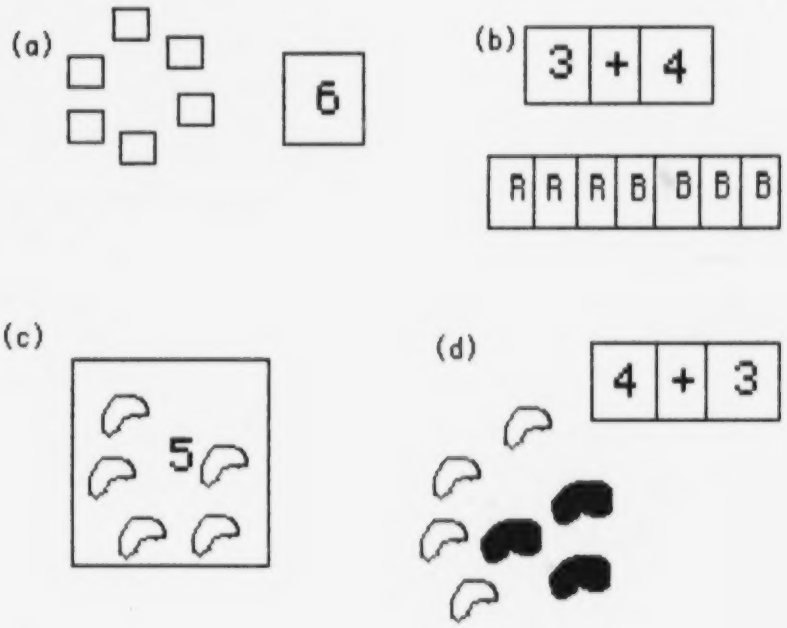

A Concept formation at the concrete level
B Connecting symbols to developed concepts
C Generating symbolic forms for configurations of materials
D Interaction between materials, pictures of these, verbal descriptions and written symbols
E Responding to written symbolic forms with arrangements of materials
F Responding to written symbolic forms with written symbols

If your assessment of how well children relate numerals to number in the concrete indicates they are still trying to establish this connection, the following lessons and activities are needed:

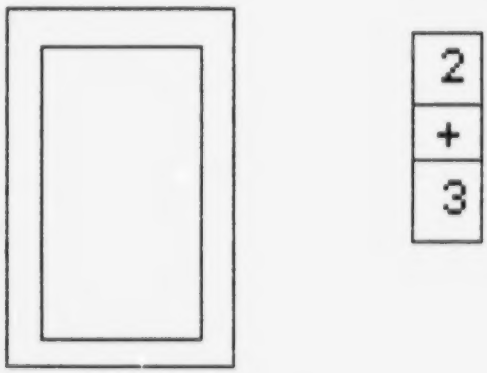
1. Prepare cards that look like those below for each number 0-9. Use these as a "flash card" activity with individual students or small groups of students.



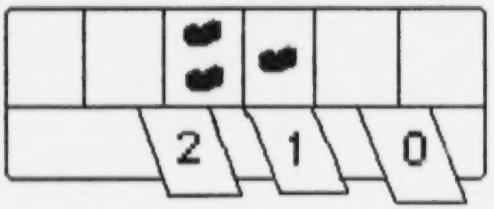
2. Prepare numeral cards with all numerals 0-9 and some with +symbols. Have children (a) place these next to collections of objects of a given number(s), arrange them to show the size(s) of the parts of a part-part-whole representation such as a two color **Unifix** cube link, (c) place on each card beans of that number, or (d) arrange to show the parts when a small handful of beans painted on one side are rolled out.




3. Use the *"Peek Through the Wall"* lesson on p. 180 of ***Mathematics Their Way***



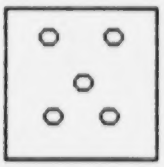
4. Use egg cartons with numbers of beans in order in the cells. Have the children place the numeral cards against the side by the cell with that number of beans.




5. Use the "*Lift the Bowl*" activity on p. 181 of ***Mathematics Their Way***.

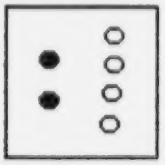
6. Have the children keep "notebooks" for each number. They can put pictures, drawings they make, records they make of work with materials and related things in these.

7. When children seem to be able to place numeral and + cards and have learned how to write the numerals, have them write number phrases to show part-part-whole relations and the cardinality of collections shown them, overhead transparencies,
(



holes cut in a card and placed on the overhead

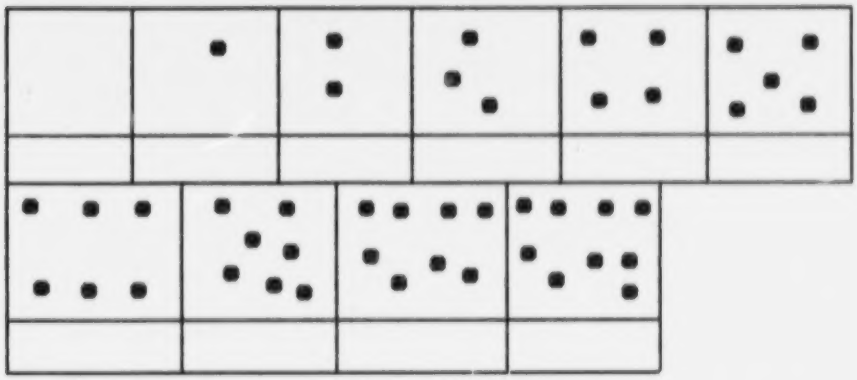
grouped material on the overhead



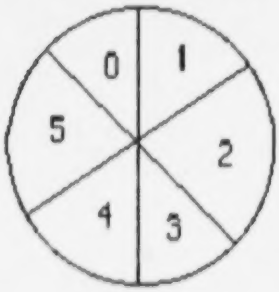
etc.

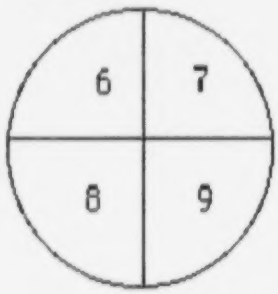
)

8. Prepare number strips for the numbers in order from 0-9:



Use spinners with the numerals on them:

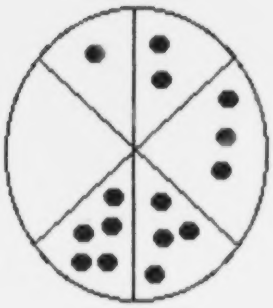


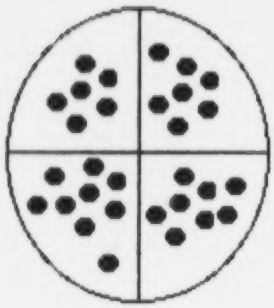




Have the children spin the pointer. They are to place that many counters in the correct square on the strip and print the numeral in the box at the bottom of the strip.

9. Prepare large (4 x 5) numeral cards. Give children counters and have them place counters of that number on each card. Make these in overhead version and as you show the numerals one at a time have children draw that many dots on their lap chalkboard. They can then hold these up for you to see.

10. Make spinners with dots on them:

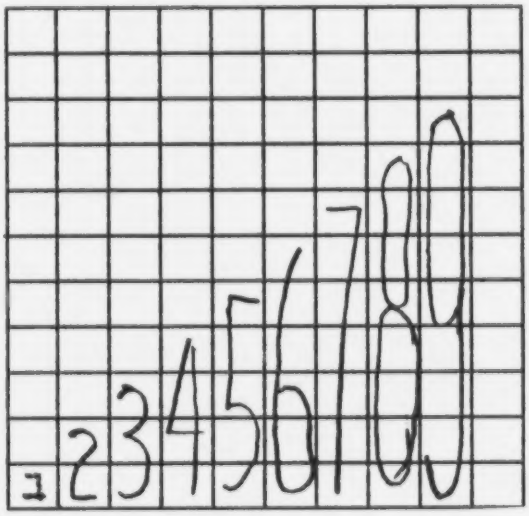




Give children numeral cards. As the spinner arrow rests on a sector, the children are to put down that numeral card.
Make overhead spinners and have the children HOLD UP the correct numeral card.

11. Place different numbers of objects such as transparency pattern blocks or counters on the overhead bed. With each display of a number, have the children hold up the numeral cards.

12. Using tagboard or poster board, prepare numeral cards so the size indicates the number.

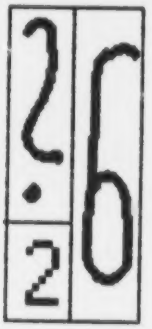


The children can be given these numeral cards and asked to:

a. put them in correct order;

b. join two together and find a third that is as long

c. take two of different length and find the card that fills the difference.

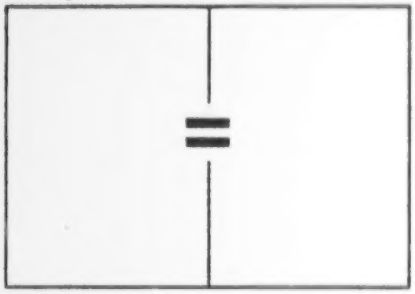


LEVEL ONE

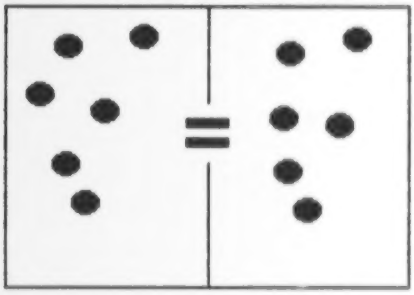
SYMBOLS FOR RELATIONSHIPS: '=' For Equality

Background: The three comparison situations between number properties of A and B are A < (less than) B, A > (greater than) B, and A = (equals) B. The first two symbols need not be introduced, or associated with MORE and LESS, in this grade. There is no need for that. The symbol for equality is more important and must be related to the concept. Children often think of "=" as meaning "here comes THE ANSWER" as the result of premature presentation of forms like 2 + 5 = _____. These lessons are designed to associate "+" with understanding of the same number property for two quantities.

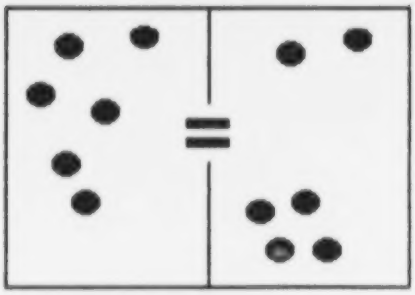
LESSON ONE: Each child should have an equality mat. This is a piece of heavy poster board, laminated if possible, that looks like:



Introduce the lesson by having an overhead transparency version of this. Place six counters on the left side and six on the right side:



Have the children verify that there are six on each side. Rearrange the right side into two groups:



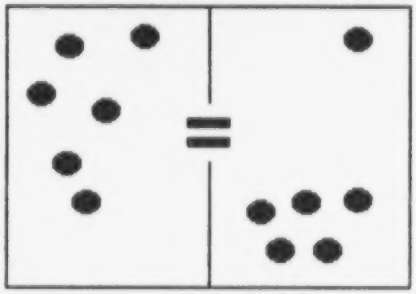
below the map place the number sentence

$$6 = 2 + 4$$

Discuss this with the children pointing out that:

1. = is used to show there are still as many on one side as on the other
2. the right side (as the children see it!) is six in 2 parts, so is 2 + 4.

Have the children arrange counters on their equality mats to show 6 = 2 + 4. Then arrange the grouped side into 5 and 1 and ask the children to write the number sentences. Point out the left side has not been rearranged. Ask for the correct number sentence to show:



Do this periodically with different numbers of counters on each of the two sides. Besides reinforcing equality this will give additional experience in expressing parts with "+".

LESSON TWO: Follow Lesson One but re-arrange the LEFT (as children see it) side and keeping the right side constant, number sentences like:

5 + 3 = 8  2 + 6 = 8

will result

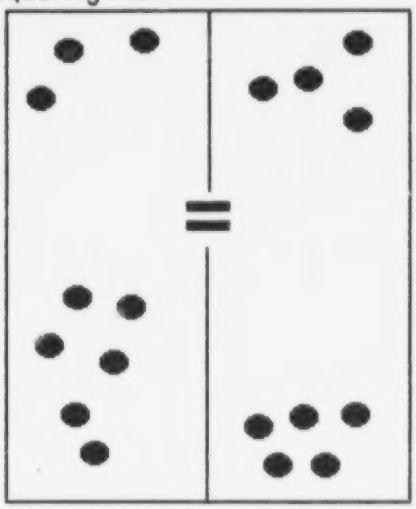
4 + 4 = 8  7 + 1 = 8

Keep both sides in ungrouped arrangement and ask the children to write one side showing parts. You may have to prompt to get:

8 + 0 = 8 and 8 = 8 + 0

Point out 0 is always a part that can be written for any number.

LESSON THREE: Follow Lesson Two but re-arrange BOTH sides of the equality mat:



6 + 3 = 4 + 5

Write the number sentence. Present several similar situations and ask the children to write the number sentences, show the number separated into parts in two ways, alike or different.

EXTENSIONS: When children are accustomed to writing the equality sentences, have the sides either one at a time or both, separated into three or more groups or one side into two groups, the other into three groups, etc. Children should become comfortable with showing such relationships as 5 + 3 = 2 + 2 + 2 + 2, etc. Coordinate this with separating quantities into equal groups by requiring equal groups on one side for even numbers or multiples of three.

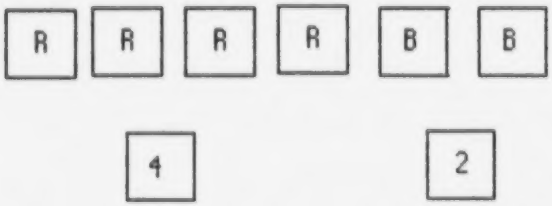
LEVEL ONE

SYMBOLS FOR OPERATIONS: + WITH PART-PART-WHOLE

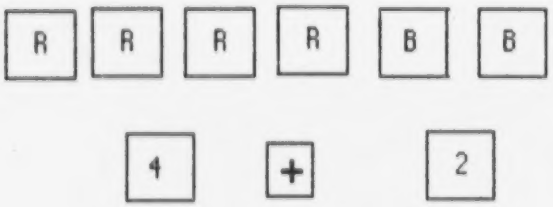
The PLUS sign, "+", and the MINUS sign, "-", can be used in several ways. "+" is used to separate the parts in a part-part- whole relation. It is also used to show the operation of joining. "-" is used to find the difference in a comparison and also to show the operation of separating. Children must understand all of these uses.

LESSON ONE: Place 6 colored squares - 4 red and 2 blue in a row on the overhead or on a flannel board.

Place numeral cards to show the size of each part.

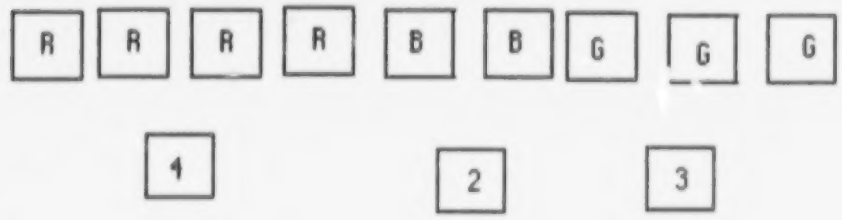



Then point out the six is PART red and PART blue and place a [+] between the numeral cards. Tell the children this sign is used to show the parts in a part-part-whole.

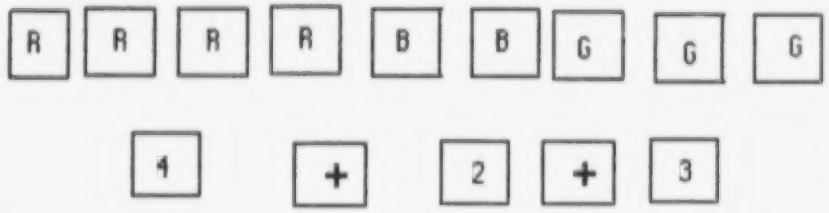



Place three different color squares on the overhead

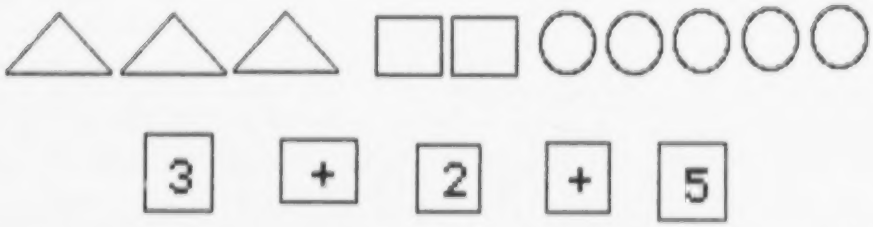
Ask the children what numeral card to put with the red squares, then the blue squares, then the green squares.




Then ask what sign must be placed between the numerals to show the line of squares is in parts.




Repeat with other sets of objects that are in parts by color, or shape.




LESSON TWO: With small groups of children who have numeral cards and "+" cards, roll out a given number of lima beans colored one color on one side and another on the other side. Have the children arrange cards to show the beans:

2 + 4

Follow both of these lessons up by having children write partial number sentences to show the part-part whole relationships. These can be written on lap chalkboards and held up so you can check them.

LESSON THREE: Make a link of 2 different color UNIFIX cubes.

| Y | Y | Y | G | G | G | G |
|---|---|---|---|---|---|---|

Ask the children to use numerals and the "+" sign to show the part of this 3 + 4

Rearrange these as shown:

| Y | Y | G | G | G | Y | G |
|---|---|---|---|---|---|---|

Ask the children to show the parts of this:

2 + 3 + 1 + 1

Have the children make links of two colors in several ways and write part-part whole phrases to show the parts of the links.

LEVEL ONE

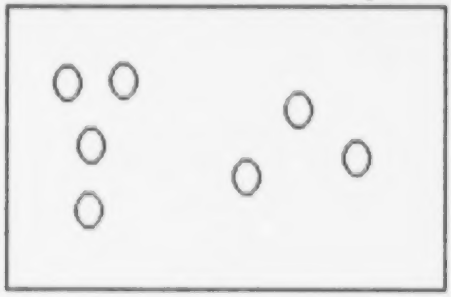
SYMBOLS FOR OPERATIONS: + WITH JOINING

LESSON ONE: Children have numeral cards and + cards and a work mat.

On the overhead projector or flannel board, place a small number of counters on a given color. Ask the children to place that numeral on the work mat. Tell them you are going to JOIN some more counters to make a bigger group. Tell them to put the + card to show that.

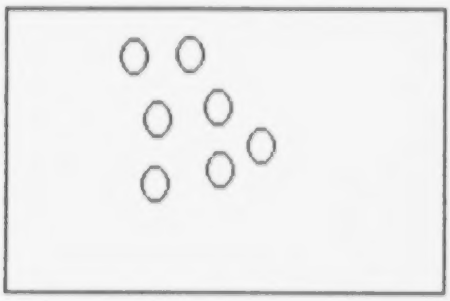
Then place another set of counters of THE SAME COLOR with those already there and ask the children to put that numeral card after the + card.

LESSON TWO: On the overhead projector or flannel board place a small number of objects of the same color or shape toward one side. On the other side place another small number of objects.

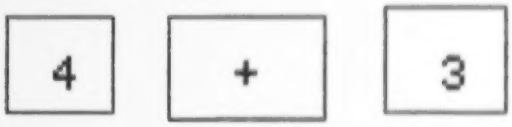


Ask the children to place numeral cards to show these two collections

then join the sets together



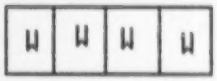
Ask the children what card shows this. Have them place that card BETWEEN the numeral cards




Ask them to hold up the card of the set that WAS JOINED to the other. Then ask them to jold up the card that was JOINED TO.

Extensions: For both of these lessons, have the children write the symbols instead of placing cards as you do the joining of objects.

LESSON THREE : Make a UNIFIX link with a small number of the same color cubes:




Make a second link of a different small number of cubes of another color:

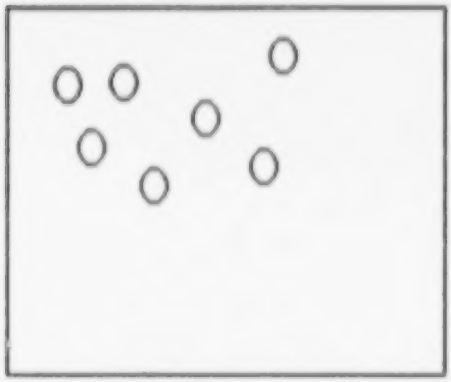
Show the children the two links, one in each hand. Tell them to write the numeral for each link. Then tell them yhou will JOIN the two links together and do it. Ask them to put the correct sign between the numerals to show the JOINING.

Give each child two colors of UNIFIX cubes. Have them make different size links and join two of these of different color together a number of times. The number phrase to show this should be written each time.

LEVEL ONE

SYMBOLS FOR OPERATIONS: '-' TO FIND THE MISSING PART

LESSON ONE: Place a collection of objects on the overhead projector or flannel board:

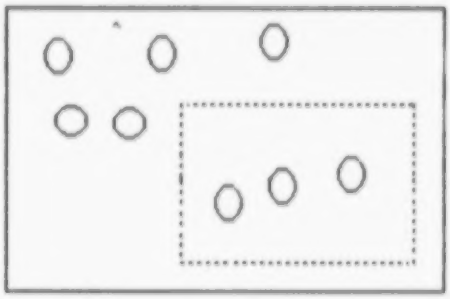


Cover up PART of this collection with a card and explain that PART is covered and PART is uncovered. Ask how they could find the covered PART knowing how many were on the overhead and how many are uncovered. Explain that "-" is used to show finding the missing part. Remove the card.

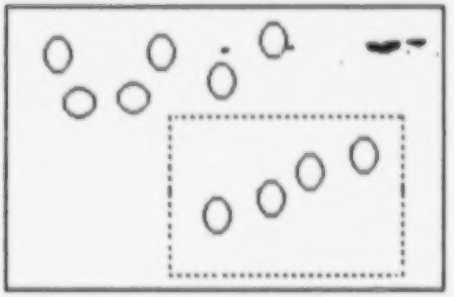
Have the children write the numeral for the number of objects seen. Then cover a subset of these and ask the children to complete the phrase using "-" and the numeral for what is now seen.

Examples:

8 - 5 for



9 - 4 for



LESSON TWO: Hold up a UNIFIX link that is part red and part blue.

| R | R | R | R | B | B | B |
|---|---|---|---|---|---|---|

Ask the children to identify the sizes of the two parts. Have them write down the numeral for the whole link. Then remove the blue part and ask them to use "-" and the numeral for the red part to show the missing blue part:

7 - 4

This (7-4) is a name for the missing part (3).

EXTENSION: Make a link of three colors:

| R | R | R | B | B | G | G | G | G |
|---|---|---|---|---|---|---|---|---|

Ask the children how many parts are there and what size they are. Break off the red part and ask them to write a phrase using "-"s and the numerals for the parts to show the missing part:

9 - 2 - 4

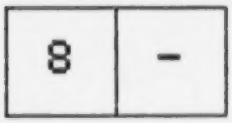
(This is a name for the missing part (3).

LESSON THREE: Show the children six or seven beans. Have them tell you how many you have. Place three or four of these under a paper cup and the remainder on top so the children can see them. Ask them how many are under the cup. Have them write a phrase using the numeral for the number of beans shown and the number of beans on top of the cup to show the number under the cup.

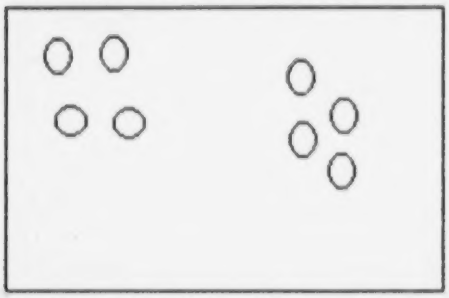
7-4 is the name for the 3 under the cup.

SYMBOLS FOR OPERATIONS: '-' FOR SEPARATING

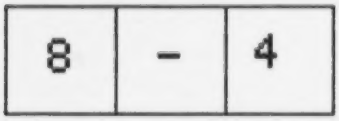
LESSON ONE: Children have cards with numerals on "-" signs. Place a collection of objects on the overhead projector. Ask the children to hold up a numeral card to show how many are in the collection and place this on the workmat. Tell them you are going to separate away some of the things and that the "-" sign is used to show this. Have them put that card behind the numeral card on their workmat:




Then separate a sub-collection of the objects:



Ask the children to place the numeral card for that number separated away after the "-".




Then totally remove the set separated out. Repeat this having the children write the expression to show "taking away" on their lapboards.

Do this with different numbers to start with and different amounts "taken away."

LESSON TWO: Hold up a UNIFIX link. Ask the children to write down the numeral for the number of UNIFIX cubes in the link. Tell them you are about to break off a link and ask them to write the sign to show this. Break off a smaller link and have them write the numeral for that number of cubes.

Repeat these until children feel comfortable using "-" to show separating away a smaller group from a larger one, and using expressions like 8 - 5 or 9 - 3 to show the number of objects left behind.

Use a variety of materials. When you separate a number of objects in equal amounts, point this out - you are dividing the number by two.

Extension: Have the children write the phrases rather than using the symbol cards. Those who need the cards should be permitted to use them until they can write the phrases correctly.

LEVEL ONE

SYMBOLS FOR OPERATIONS: '-' FOR FINDING THE DIFFERENCE

LESSON ONE: Children are to have cards with numerals and "-" signs. On the overhead projector or on the flannel board place two rows of counters - one row of one color, the other row of another color:

Ask the children which row has MORE, then to show a numeral card for the number in that row. Then ask them to show a numeral card for the number in the row with LESS. Point out that "-" is used to show how many MORE are in the longer row, or how many LESS are in the shorter row. Have them place the "-" card between the numerals:

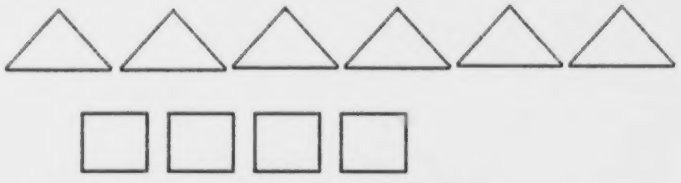
| 8 | - | 5 |
|---|---|---|

This represents HOW MUCH MORE the longer row has.

Emphasize the idea that "-" is used to COMPARE the two amounts of things.

LESSON TWO: Children should have chalk lapboards. Hold two unequal UNIFIX cube links up so the children can see them. Ask them to write the numerals and "-" sign to show how many MORE cubes the longer link has, or how many FEWER cubes the shorter link has. Have them hold up the cards. They should have "7 - 4" or "8 - 5" or similar expressions on the chalkboard.

Extension: These lessons should be repeated with different amounts and different kinds of materials until children are comfortable showing THE DIFFERENCE by "a-b" expressions. Pattern blocks, cut out geometric shapes and similar materials can be used:



"Are there MORE triangles or squares?"
"How many more triangles than squares?"

"How many fewer squares than triangles?"
Keep emphasizing the idea that the DIFFERENCE can be expressed using "less than" when referring to the smaller or shorter and "more than" when referring to the larger or longer.

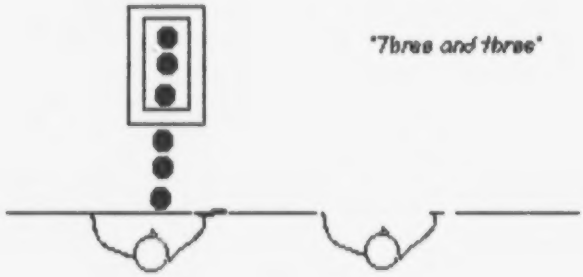
NUMBER OPERATIONS: GAMES

*The Combine Game (for two children):* Needed are counters, a window frame cut from cardboard and numeral and operation sign cards.

The number of counters used is that number the two children are working to internalize.

The counters are placed in as line before Child A, who then uses the frame to separate the counters into two parts. Child B is to identify the part-whole combination made first (1) orally, then (2) using symbol cards.

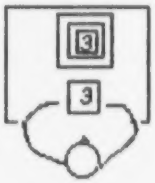
Example: Working on "SIX"



*"Three and three"*

*Step 1.*

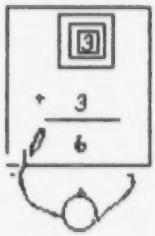
*same*




*"Three and three" as numeral cards are placed on recording form.*

*Step 2.*

*same*



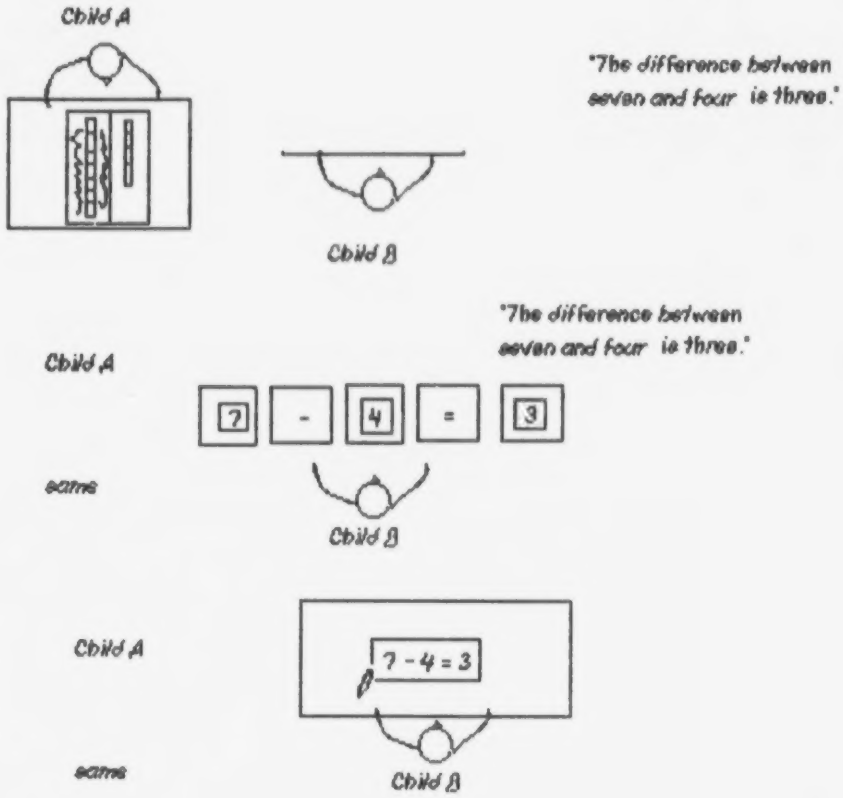

*Writes on recording form*

*Step 3.*

*Child B*

*The Difference Game (2 children):* This uses a place value mat and cards with numerals and operation and = signs.

Child A prepares two links of the same length in UNIFIX cubes, i.e. 7, places one on the colored half of the mat, breaks a few cubes off the other and places it on the white half. Child B expresses the difference orally, then places the numeral and other cards, then writes the number sentence:



LEVEL ONE

NUMBER OPERATIONS: GAMES 2

*The Take Apart Game (2 players):* Used are a given number of counters or UNIFIX cubes, numeral and sign cards.

Example: Number 7

Child A places the given number of counters or UNIFIX cube link, on the table. Child B covers eyes and Child Aeither covers and moves some number of counters or breaks off and hides a small link of UNIFIX cubes. Child B uncovers eyes and describes what occurred:

*Child A*

*Child B*

*"You took away five from seven and have two left"*

*Step 1*

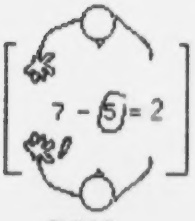
*Child A*

*Child B*

*"You took away five from seven and have two left"*

*Step 2*

*Child A*




*Child B*

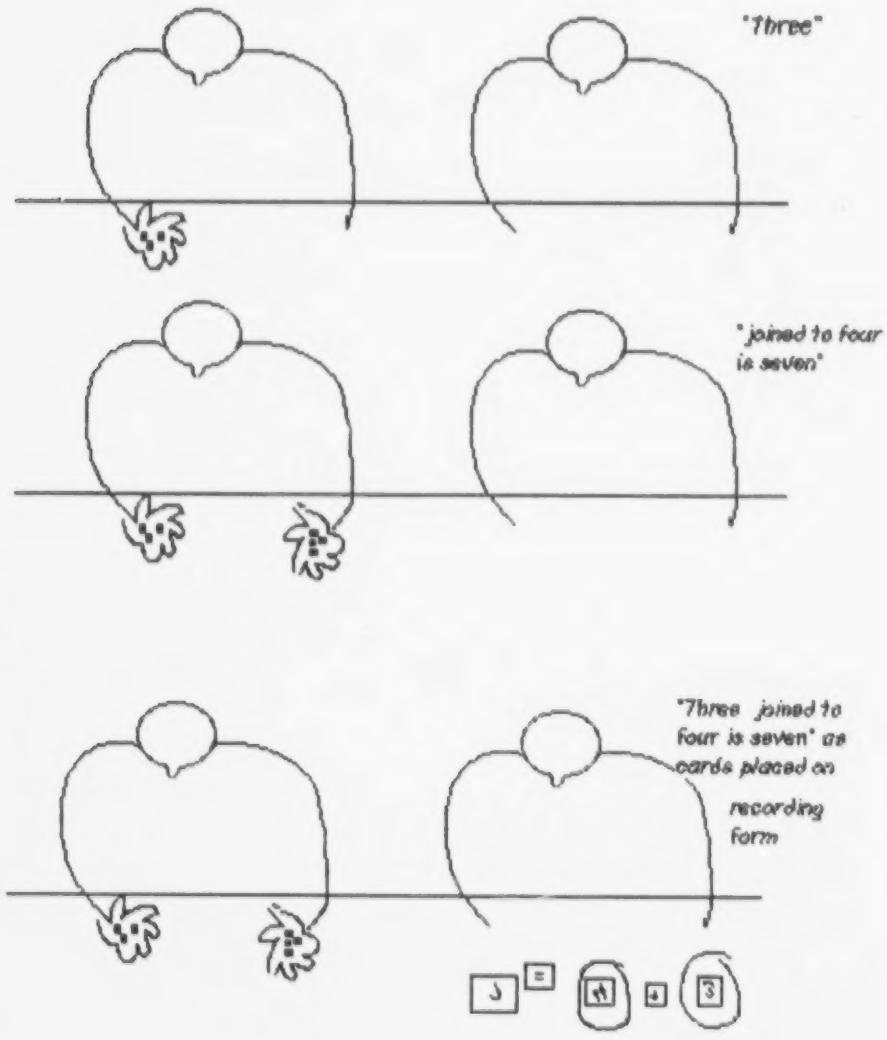
*"You took away five from seven and have two left"*

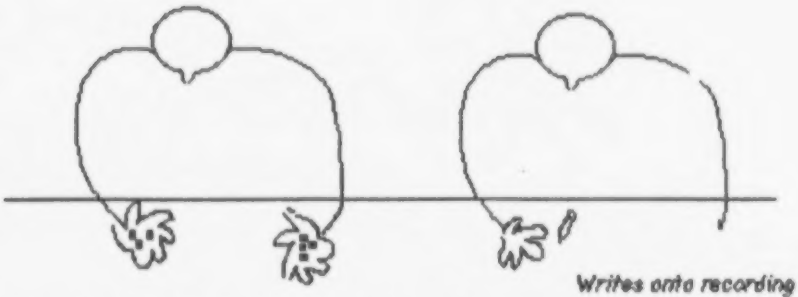
*Step 3*

*The Put Together Game (2 players):* This game uses counters or UNIFIX cubes, numeral sign cards and recording forms.

Child A shows a part of the number being worked on in the left hand, then shows the other part in the right hand. Child B describes what happens orally, uses numeral cards to show it, and finally writes the number sentence to show it.

Example: Number is 7






With Unifix: Child A shows one link, then second link, then links the two together. Child B orally describes joining, then places numeral cards, then writes number sentence.

LEVEL ONE

NUMBER CONCEPT: Writing Number Sentences

Background: Once children understand how expressions like "a + b" and "a-b" are used to indicate numbers associated with part-part whole relations, joining, separating and finding the differences, and that "=" shows the same number on both sides, they are ready to write number sentences like 6 = 2 + 4, 8 - 3 = 5, etc.

LESSON ONE: (Part-Part Whole)

Show children a link of UNIFIX cubes:

| R | R | R | R | R | G | G | G | G |
|---|---|---|---|---|---|---|---|---|

Ask them how to show the parts. They should respond with "5 + 4" if past lessons have been successful. Write this on the board. Then ask them how many UNIFIX cubes in the link. They should respond with "nine." Point out these are two names for the same number of cubes and complete the sentence as:

5 + 4 = 9

Ask the children if it makes any difference which name is on which side of the "=" and write:

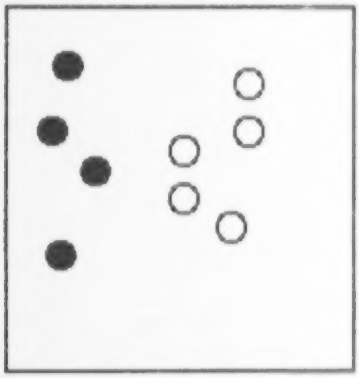
9 = 5 + 4

Take time to make sure the children realize these two sentences show the same situation. Have the children make several 2 color UNIFIX links of different numbers of cubes and different size parts. They should write TWO number sentences for each:

8 = 2 + 3 + 2 + 1
2 + 3 + 2 + 1 = 8

LESSON TWO: (Part-part-whole)

On the overhead projector place two colors of counters similar to the following:



Ask the children to write TWO sentences to show the number of counters on the projector.

4 + 5 = 9 and 9 = 4 + 5

Point out the parts could be written another way and write:

5 + 4 = 9 and 9 = 5 + 4

Give the children five lima beans painted a different color on each side. Have them roll these out and write what they see FOUR different ways:

3 + 2 = 5 2 + 3 = 5

5 = 3 + 2 5 = 2 + 3

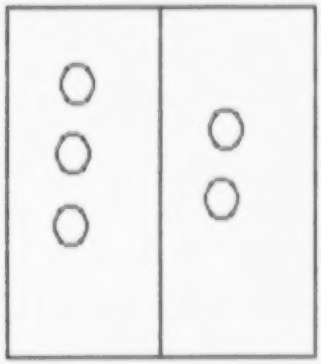
Have them roll the beans ten times writing 4 sentences each time.

Introduce the recording forms at the number stations that have space on them for writing number sentences.

LESSON THREE: (Joining)

Children should have work mats and counters of some type, but all of the same color, e.g. all green UNIFIX wooden cubes, red plastic chips, etc.

Tell the children to put 3 chips on the left side and 2 chips on the right side.



Tell them to join the right side to the left side and show this with a number sentence showing the joining and the total number of counters:

3 + 2 = 5

Have them put the 2 counters back on the right side. Tell them to join the left side counters to those on the right, and write the number sentence to show the joining and the total number of counters:

2 + 3 = 5

Ask them if 5 = 2 + 3 would be O.K. Point out the total number can't always be seen until after the joining, so 2 + 3 = 5 better represents this.

Repeat this several times with different numbers on the two sides and different total numbers of counters.

LESSON FOUR: (A Joining Story)

"Peter had 4 crackers. Put UNIFIX cubes on your workmat to show Peter's 4 crackers. His mother gave him 3 more crackers. Join 3 UNIFIX to those on the work mat. Now write the number sentence to show 3 joined to 4 and the resulting number."

4 + 3 = 7

Write the correct number sentence on the board and point out:

"4" is what you started with.
"+" shows the joining.
"3" is what is joined.
"=" shows the same number on both sides.

"7" shows the result of the joining.

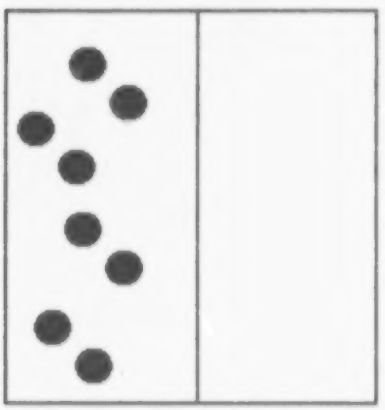
Create four or five more "JOINING" stories and have the children:

(1) show what happens with the cubes, and

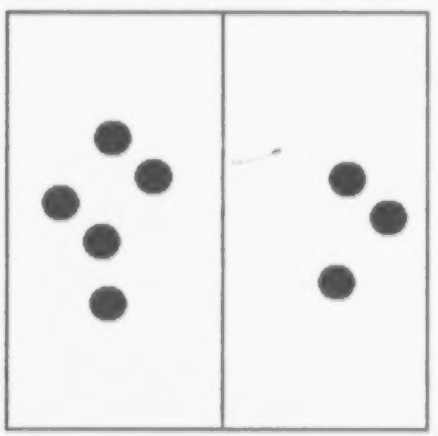
(2) write a number sentence to show what happened.

LESSON FIVE: (Separating)

Children should again have work mats and counters. Have the children put eight objects on the left side:



Tell them to take 3 away from the left side and put these on the right side.



Have the children write a number sentence to show this separating.

8 - 3 = 5

"8" is what you start with.
"-" shows separating.
"3" is what is "taken away".
"=" shows the same number on both sides.
"5" shows what is left.

Give them several different numbers to start with and differing amounts to separate out. Have them write number sentences to show each separating.

LESSON SIX: (A Separating Story)

"Nancy had 9 crayons. She gave 4 of them to Gail and had 5 left."

"Show this with UNIFIX cubes and write the number sentence."

Write the sentence on the board and point out all of the symbols and what they mean:

9 - 4 = 5

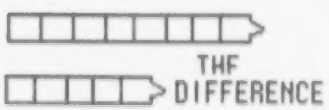
Create four or five simple "take away" or separating stories and have the children:

(1) show what happens with counters, and

(2) write the correct number sentences.

LESSON SEVEN: (Comparing)

Have the children make 2 links of UNIFIX cubes - one with 8 cubes and the other with 5 cubes. They should lay these along side each other to facilitate seeing "the difference."




Have them write a number sentence to show the difference.

8 - 5 = 3

"8" is the larger.

"-" shows finding the difference
"5" is the smaller
"=" shows the same number on both sides.
"3" shows "the difference" between 8 and 5.

8 is 3 MORE THAN 5 and
5 is 3 LESS THAN 8.

Have the children make several pairs of links of different numbers and write number sentences to show "finding the difference" between them.

LESSON EIGHT: (A Comparing Story)

"Joan had seven books. Her brother had four books. Show Joan's books and her brother's books with UNIFIX cubes.....Joan had how many more books than her brother?.....Her brother had how many books fewer than Joan?" Have the children write the number sentence to show this:

$$7 - 4 = 3$$

**the difference** **the size of the difference**

Create other simple comparison stories. Have the children represent the objects in the story with materials. Use both kinds of questions. Have the children write the number sentences.

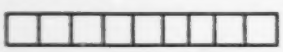
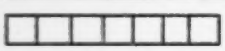
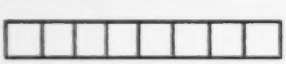
NUMBER CONCEPT: (Addition and Subtraction)

As children develop each number (such as six) integrate this with the use of + and - signs by having them do the following kinds of activity.

LESSON ONE: With Unifix Cubes:
A number is assigned. Eight (8) will be used as an example.

Each child makes a link of two cubes. Assign the number of signs (+ and -) to use in reaching 8 (4 will be used as an example). The goal is to start from 2, add and subtract cubes to use the required number of signs and have a final result of 8.
Example:

(1) (2) (3) (4) (5)
2 + 4 + 3 - 2 + 1 = 8

(1)
(2)
(3) 
(4) 
(5) 

Ask each child to start from 2, use 4 signs and finish with 8 ten different ways.

Extensions: Change any one of the three
(1) starting number
(2) number of + and - signs used
(3) the ending number
and you will have a new activity. These can be repeated throughout the year.

LESSON TWO

Pass out numeral cards with symbols 0 through 9 on them, one to each child. Ask the children to hold up cards in response to questions like:
*"Who is four plus one?"*

*"Who is three minus two?"*
*"Who is six plus two?"*
*"Who is five minus three?"* etc.

LESSON THREE

Introduction: *"Guess My Rule"* can be played using several concepts - number, numeration, shapes, colors, etc. In this lesson are several activities that involve two numbers that are related in some way.

You tell the children you have a rule to change the numbers they give you into new numbers. They are to, one at a time, give you small numbers less than ten. You then apply your rule and give the related number. Children are to guess the rule. Example 1:

Child 1: "three"
You: "five" - Write each pair in the table
Child 2: "two"
You: "four"

TABLE

The rule is *"two more"* or *"add two"*

| | IN | OUT |
|---|---|---|
| 1) | 3 | 5 |
| 2) | 2 | 4 |
| | etc. | |

Example 2:
Child 1: "four" Write each pair into the table
You: "three"
Child 2: "six"
You: "five"

TABLE

The rule is *"one less than"* or *"subtraction"*

| | IN | OUT |
|---|---|---|
| 1) | 4 | 3 |
| 2) | 6 | 5 |

When children are comfortable playing this with a variety of rules, give them the worksheet to (1) complete, and (2) write the rule.

LESSON FOUR:

Put a table on the overhead or chalkboard:

| IN | OUT |
|---|---|
| | |
| | |
| | |
| | |
| | |

Put a numeral in the "IN" column:

| | IN | OUT |
|---|---|---|
| 3 | | |
| | | |
| | | |
| | | |

The rule is: *"Write the next number; What should I write?"*
Put another numeral in

| IN | OUT |
|---|---|
| 3 | 4 |
| 8 | |

Ask another child to furnish *"the next number."*

Extensions: Use *"Guess My Rule"* with geometric shapes - Attribute Blocks, Pattern Blocks, Geoblocks
"They are the same color"
"They are both squares"
"They are both small squares"
"Both blocks have triangles on them"

UNIFIX Cube links: "They are both six"
"They have the same pattern" - etc.

LESSON FIVE:

Pick a "number for the day" such as five. Have the children write as many expressions for five as they can think of. Give them UNIFIX cubes to help them generate these.

Example: Five
2 + 3, 3 + 2, 1 + 4, 4 + 1, 5 + 0, 0 + 5, 2 + 2 + 1, 6 - 1, 7 - 2, 3 + 1 + 1, etc.

LESSON SIX

Give the students "What Do I Have To Do" stories.
"I have six. I need eight."
"I have seven. I need only four." etc.

LEVEL ONE

DECODING NUMBER SENTENCES

Background: Once children are comfortable with WRITING number sentences to show arrangements of materials and the conditions in orally stated situations, it is time to reverse the process and have them decode number sentences.

LESSON ONE: (+ to show part-part whole)

This lesson involves the canonical form $c = a + b$ or $a + b = c$.

The children should have UNIFIX cubes of 2 different colors and WORKMATS.

Introduce this by writing a number sentence on the board, e.g.:
$6 + 3 = 9$.

Ask the children to make a UNIFIX link to show what this means as a part-part whole relation. Have them hold up the links formed for you to check. Remind them of this interpretation of the "+" sign and the role of the numerals and "=" sign.

Write several similar sentences, alternating between $a + b = c$ and $c = a + b$. Have the children make part-part whole UNIFIX links for each.

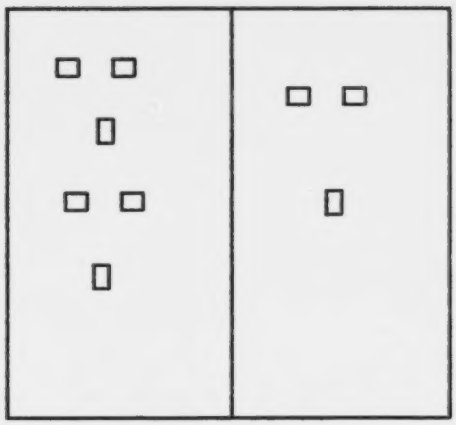
Pass out WORKSHEET 1 and have the children complete this. Review the worksheet directions with them.

LESSON TWO: (+ to show joining)

The children should have UNIFIX cubes of the same color and split WORKMATS.

Write a number sentence on the board: $9 = 6 + 3$

Have the children place six cubes on one side of the board and three cubes on the other side.



Ask the children what the "+" means when you have two groups. Have them join either the 3 to the 6 or the 6 to the 3 to get 9.

Write several similar number sentences of both forms a + b = c and c = a + b.

Have the children place the given quantities on the two sides of the mat and then join them together. Observe what they do and frequently emphasize the roles of the numerals, + and =. Pass out WORKSHEET 2 and have the children do this. Go over the directions with them first.

LESSON THREE: (- to show a difference)

Children should have UNIFIX of one color. Write a number sentence on the board, e.g.: 9 - 6 = 3

Tell the children to make a 9 UNIFIX link and a 6 UNIFIX link and lay them side by side. Remind them the 3 shows the difference and the "-" means to find the difference. Tell them that the number sentences you write with "-" in them are to represent this kind of comparison.

Write several sentences of the form a - b = c and c = a - b wuch as:

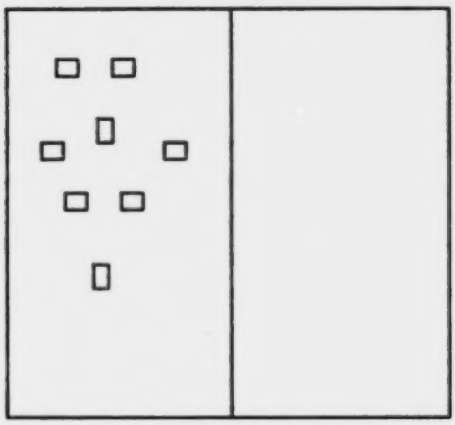
8 - 3 = 5 and 4 = 7 - 3

The children are to make UNIFIX cube links to show each of the two quantities being compared. When it appears they are comfortable use WORKSHEET 3.

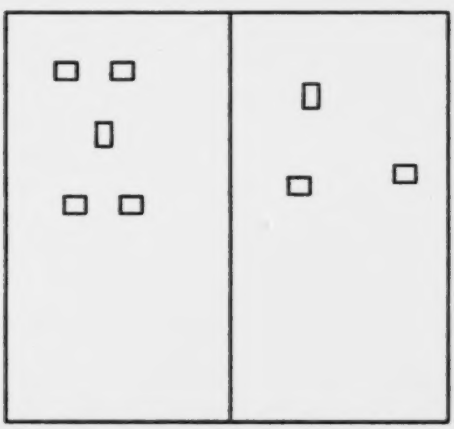
LESSON FOUR: (- to show separating)

Children should have UNIFIX cubes of the same color and a split WORKMAT.
Write a number sentence on the board: 8 - 3 = 5

Have the children put 8 cubes on one side of the mat:



Remind them "-" can mean separating away a smaller group. Have them separate what is shown in the sentence from the 8 and put it on the other side. Ask them to see that the 5 shows what is left from the 8:



Write several number sentences of the form 4 = 9 -5 and
7 - 2 = 5 on the board.

The children are to make the large quantity on one side separate away the smaller quantity and observe what is left. Circulate and observe this activity. When the students are comfortable with this interpretation of the "-", give them WORKSHEET 4.

Repeat these as needed until children can decode number sentences

into part-part whole, joining. separating and comparing situations.

Extension: Use WORKSHEET FIVE and others like it you devise as practice in decoding number sentences. Children should be given UNIFIX cubes and workmats so they can interpret "+" and "-" as either part-part whole or joining, and separating or comparing.

LEVEL ONE

## INTRODUCING WORD PROBLEMS

Introduction: Children must know which operations are suggested by language used. Actions such as joining and separating are suggested by certain language, while static situations involving part-whole and comparison are suggested by other language.

Children should be introduced to these uses with problems that have no given numbers in order to concentrate on the meaning of the language.

LESSON ONE: Children should have lap chalkboards and UNIFIX cubes. They are to write the operation symbol, "+", or "-", to show what they think should be done with the given quantities in the problem on the lapboard and hold it after you have read the problem.

Read each problem slowly to the children. Observe the operation symbols displayed and discuss each problem to find out why the children are writing each operation sign.

Problem One: "Charlie found some nuts. Jane gave him some more nuts. How many nuts did Charlie have then?"

This is a simple joining and + shows the operation to be performed on the numbers had same been given. The form is:

A + B = ☐ or ☐ = A + B

LESSON TWO: "Bobbie had some marbles. He gave part of his marbles to Jackie. How many marbles did he have left?"

This is a simple separating and "-" shows the operation linking the two given quantities. The form is:

A - B = ☐ or ☐ = A - B

LESSON THREE: "Billie had some soda pop straws. His sister gave him some more. I know how many Billie had then. How many did his sister give him?"

This is a missing addend situation:

A + ☐ = B

Since it is a joining, some children will write "+". Others may realize what must be done with the given A and B and write "-". Ask the children to explain why they wrote what they did and discuss these differences.

LESSON FOUR: "Charlie Chipmunk gathered some nuts. He gave part to his sister and had some left. I know how much he had left. How many nuts did Charlie give to his sister?"

This is a separating of the type:

A - ☐ = B

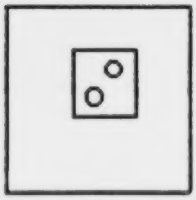
The separating is shown by "-" and "-" is performed on the given quantities so either way the children should show you "-".

LESSON FIVE: "Corrine found some berries on a plant and put them in a pail. Paul gave her his berries to put in the pail. I know how many berries were then in the pail. How many berries did Corinne have to start with?"

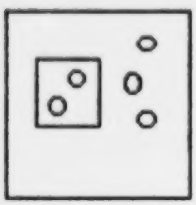
This is of the form:

☐ + A = B

"+" shows the joining and "-" shows what to do with A and B. This will require extensive discussion, with some small numbers for examples. Perhaps the use of chips on the overhead would be needed:



Add three chips:



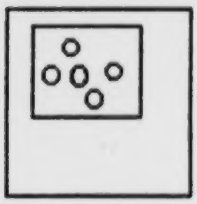
"Now there are five chips. How many are under the card?"

LESSON SIX: "Polly had some stuffed animals. She gave part of these to her sister and counted how many she had left. How many did she have to start with?"

This is a separation of the form:

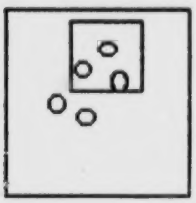
$\square - A = B$

The separation is shown by "-". "+" shows what to do with the known quantities, so children again might display either sign. Discuss this. Do an example on the overhead if needed.



Chips under a card

Lift the card slightly and remove a few



Lift the card to show how many are now there and ask the children how many were under the card at the beginning.

LESSON SEVEN: "Susie Squirrel found some red leaves and some brown leaves for her nest. How many leaves did she find altogether?"

This is a simple part-part whole with NO ACTION indicated. The form is:

A + B = ☐

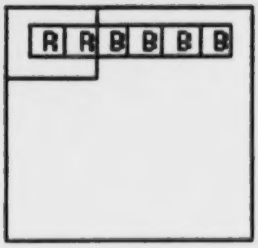
Children should show a + to indicate combining the parts to find the whole. If need be, show a 2 color UNIFIX link and compare this with the language in the problem.

LESSON EIGHT: "Charlie Chipmunk had some red berries and some blue berries. I know how many red berries he had and how many berries he had altogether. How many blue berries did Charlie have?"

This is a part-part whole where a missing part must be found. If children fully understand + separate parts, they will write "+". Some may write "-" to show what to do with A and B in:

A + ☐ = B

Again you may have to make a row of chips on the overhead:




Cover the red with a card before showing the display. "Some of the row of chips are red and some are blue. I have six chips in all. How many of the chips are red?"

☐ + 4 = 6 shows this, but 6 - 4 will give the ☐ value.

LESSON NINE: "While picking berries, John saw some bears. Patty saw fewer bears than John. How many more bears did John see?" This is finding the difference in a comparison and [-] will show this.

A - B = ☐

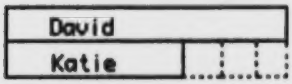
If necessary show a red cube link and a shorter blue cube link to model this.

LESSON TEN: "There were some robins in a tree. In another tree were some bluebirds. There were more bluebirds than robins. There were how many fewer robins?"

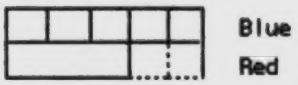
This is another find the difference comparison (A - B = ☐), but with the opposite comparison language. You will have to really spend time contrasting these two situations.

LESSON ELEVEN: "Katie has some flowers. David has 3 more flowers than Katie. I know how many David has. How many flowers does Katie have?"

This is a comparison where the difference is given and the smaller must be found so - will show this. You may have to model this with UNIFIX links.




"There are 5 blue cubes and 2 more blue cubes than red cubes. How many red cubes are there?"




LESSON TWELVE: "Freddie Fox put some leaves in a pile. Frannie Fox had 4 fewer leaves in her pile. How many leaves are in Frannie Fox's file?"

This again gives the difference (4) and the larger quantity, so - is the operation to find the smaller quantity. Again you may have to give several examples with "more than", "fewer", "less than" language.

LESSON THIRTEEN: "Tony had some boxes. He had 3 more boxes than Tom. I know how many Tom has. How many does Tony have?"

In this comparison the difference and the smaller quantity are given, so + is needed to find the larger quantity. Use UNIFIX links and several additional examples to distinguish this from the two previous cases.

LESSON FOURTEEN: "Susie Squirrel has some sunflower seeds. She has two fewer than Charlie Chipmunk. I know how many Susie has. How many does Charlie have?"

Again, the difference and the smaller are given, so + is used to find the larger. The difference is in how the comparison is stated A compared with B, or B compared with A..

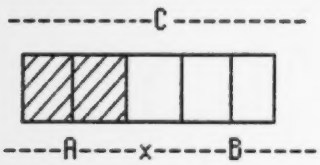
Those problems in Lesson One, Lesson Two and Lesson Seven have proved to be easiest for children to interpret correctly. Those in Lesson Three, Lesson Six, Lesson Nine, and Lesson Fourteen have proved to be the most difficult for children just beginning school and well into third grade, when reversibility of thinking is more likely to be found. Work harder on these more difficult situations for the children.

LEVEL ONE

MORE WORD PROBLEMS:

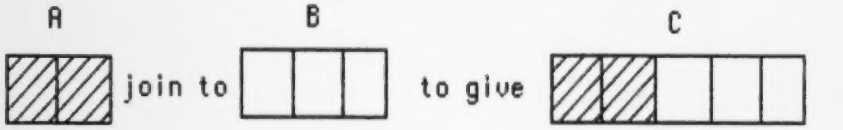
Background: All arithmetic operations are based on joining, separating, recognition of part-part-whole and comparing. Some joinings involve questions that require subtraction; some separatings involve questions that require addition, etc. There are 14 basic and different addition and subtraction types that result from using two quantities, either equal or unequal.

Part-part-whole



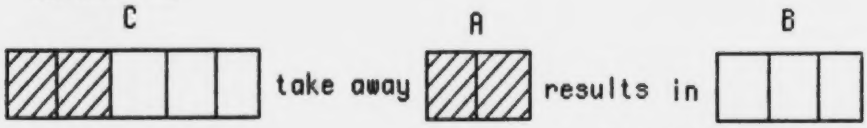

| Known | Wanted | Sentence | Operation Needed on Known Quantities |
|---|---|---|---|
| A,B | C | □ = A + B | Addition |
| A(or B),C | Other Part | C = □ + B | Subtraction |

JOINING



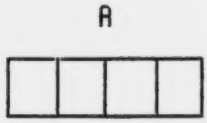

| Known | Wanted | Sentence | Operation Needed on Known Quantities |
|---|---|---|---|
| A,B | C | A + B = □ | Addition |
| A,C | B | A + □ = C | Subtraction |
| B,C | A | □ + B = C | Subtraction |

SEPARATING

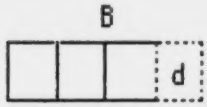



| Known | Wanted | Sentence | Operation Needed on Known Quantities |
|---|---|---|---|
| C,A | B | C - A = □ | Subtraction |
| C,B | A | C - □ = B | Subtraction |
| A,B | C | □ - A = B | Addition |

COMPARING




1) "less than" language used to describe "d".




2) "more than" language used to describe "d".

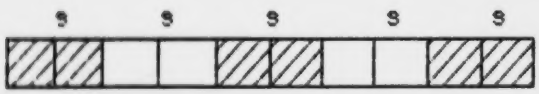
| Known | Wanted | Sentence | Operation Needed on Given Quantity |
|---|---|---|---|
| A,B | d | A - B = d | Subtraction |
| A,d | B | A - □ = d | Subtraction |
| B,d | A | □ - B = d | Addition |

3 for each of (1) and (2), totalling 6 cases.

There are 16 basic and different multiplication and division types. Those that are primarily part-part-whole, joining, separating and comparing activities involving EQUAL groups being joined and separated are described below.

Part.....Part-whole

several
(equal parts)

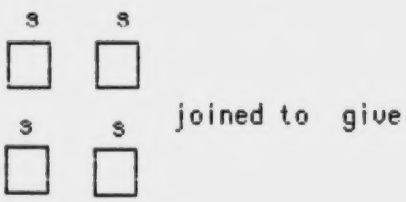


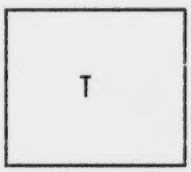

T is the whole of these equal parts

n of these

| Known | Wanted | Sentence | Operation Needed on Given Quantities |
|---|---|---|---|
| s,n | T | T = s x n | Multiplication |
| s,T | n | T = s x □ | division |
| n,T | s | T = □ x n | division |

JOINING (several equal sets)

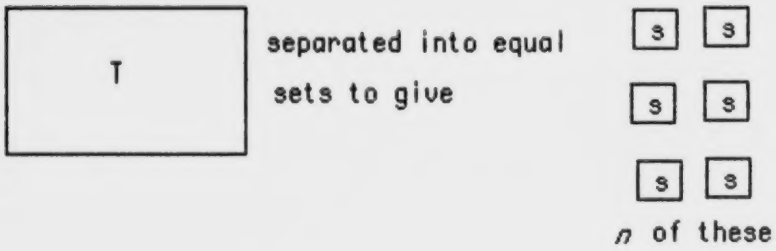



*n* of these

| Known | Wanted | Sentence | Operation Needed on Given Quantities |
|---|---|---|---|
| s, *n* | T | T = s+s+s+...s | Multiplication |
| | | or | |
| | | T = s x n | |
| s,T | *n* | T = s x □ | division |
| n, t | s | T = □ x *n* | division |

Separating (into several equal sets)




| Known | Wanted | Number Sentence | Operation Needed on Given Quantities |
|---|---|---|---|
| T,s | n | T = s x □ | division |

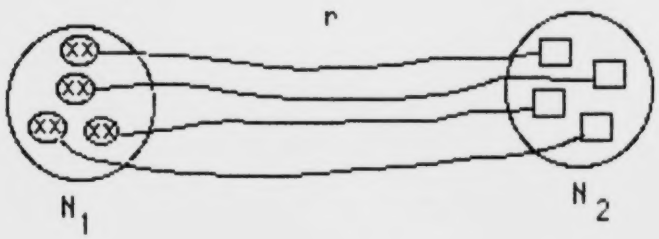
This is the "measurement" interpretation of division - T is measured by known s

| | | | |
|---|---|---|---|
| T,n | s | T = □ x n | division |

This is the "partitive" interpretation of division - T is separated in "n" sets of unknown size

| | | | |
|---|---|---|---|
| n,s | T | □ = n x | multiplication |

Comparing




r is RATIO of x x's to □ 's
or correspondence of x's with □ 's

`1

| Known | Wanted | Number Sentence | Operation Needed on Given Quantities |
|---|---|---|---|
| $N_1$, $N_2$ | r | $\square = \frac{N_1}{N_2}$ or $N_1 : N_2$ | division |
| $N_1$, r | $N_2$ | $r = \frac{N_1}{\square}$ or $N_1 : \square$ | division |
| $N_2$, r | $N_1$ | $r = \frac{\square}{N_2}$ or $\square : N_2$ | multiplication |

Children should have experience with all of these situations that involve part-part-whole, joining, separating and comparing both unequal and equal quantities.

Have them arrange the materials used in response orally given situations. Emphasize what language indicates comparison or looking at existing part-part-whole and language that indicates actions of joining and separating.

EXTENDED STORY PROBLEMS (NO WRITING)

PROCEDURE: Children should have UNIFIX cubes to represent the objects. After each question, give the children time to manipulate the objects. Then call on one child to give the answer orally.

*WINNIE THE POOH:*

Winnie the Pooh and Eeyore were having a party (2). Then Christopher Robin came. How many were at the party ? (3) Pretty soon, Tigger came pouncing in. Then how many were there? (4) Christopher Robin heard his mother calling him, so he had to leave. Then how many were at the party? (3) In short time, Owl, Rabbit and Piglet came. Then how many were at the party? (6) The animals wanted to sit at the big table to have some food. Three of them sat on one side of the table. How many sat on the other side? (3) Four of the animals had cake and 2 of them had cookies. How many more had cake than cookies? (2) Pooh asked how many would like some honey. He was the only one who wanted honey. How many did not want honey? (5) When Tigger was finished eating, he got so bouncy he had to bounce away. How many animals were still at the party? (5) Rabbit and Piglet were getting very tired, so they decided to go home. How many were at the party then? (3) One of the animals wanted to play a game, but the others did not. How many did not want to play? (2) Owl decided he should fly back to his tree, so he said goodbye and flew away. How many were there then? (2) The party of Winnie was over, so Pooh and Eeyore went home. How many were at the party then? (0)

*HALLOWEEN:*

Winnie the witch was getting ready for Halloween. First, she decided to make her happiness spell, so everyone would have a good time. She got her pot and filled it with water Then she added 2 spider legs and 3 webs. How many things are in her pot? (5) After that, she added 2 moonbeams. Now how many things has she put in her pot? (7) Next she added 2 furry bugs and 1 frog. Now how many things has she put in her pot? (10) She stirred and stirred, while chanting magic words. Suddenly the frog hopped out of the pot and hopped away. Now how many things are in her pot? (9) Then the 2 moonbeams streamed out of the pot and went back into the sky. Now how many things does her pot have? (7) Then the 2 furry bugs and the 3 webs danced out of the pot. So now, Winnie only has _____ things in her pot. Last, the 2 spider legs hopped out and went away. Now her brew was ready, so she set it aside. Soon some witches came. Three of them came on

brooms. Now there were 6 witches at the party. How many did not come on their broom? (3) She only had 4 turtle seats, so how many had to sit on the floor? (2) Winnie gave 2 of them a glass of happiness brew and they started to cackle. How many were not cackling? (4) These four witches wanted to be happy, too. So, Winnie gave them the brew. Soon 3 witches jumped on their brooms and went cackling into the air. Now how many witches are at the party? Soon it was midnite and the rest of the witches went home. Winnie was happy with her party.

*BEN FRANKLIN:*

Ben Franklin was so busy inventing things that he sometimes needed some help keeping count of all the things he had. Let's see if we can keep count of what he had.

There was a time when he got very tired of sitting in chairs that were uncomfortable. So he got wood and made 1 rocking chair. His friends liked it so much, he made 5 more.

How many did he make? (6) Then he gave 2 chairs to his friends. How many did he have then? (4) He kept giving them away until he only had 1 rocking chair for himself. How many did he give away? (3) He liked his chair very much. Soon after, he decided to move, and he packed all of his furniture on a big wagon. As the horses pulled the wagon, it hit a big bump in the road, and the rocking chair fell off the wagon and broke into many pieces. How many rocking chairs did he have then? (0)

Ben Franklin also liked to read books. He thought everyone should have books to read, so he began to lend his books to people. He lent 4 books to George Washington, and 5 books to Thomas Jefferson. How many books did he lend? (9) They returned 6 of the books very soon. How many did they still have? (3) A week later, they returned the rest of the books. How many did they have then? (0) Ben Franklin thought that lending books was such a good idea, he started the first public library.

*ST. PATRICK'S DAY (Using Lucky Charm Cereal):*

Loopy, the leprechaun, was a happy person. He loved to hop and skip through the woods. One day when he was skipping through the woods, he found a pile of Lucky Charms. He was delighted..."I wonder how many there are in this pile," he thought. How many do you think are in your pile? (Let each child estimate.) Then he saw that there were 2 kinds of charms, some were brown and some were colored. He wondered if he had more colored charms or brown ones. Look at your pile...What do you think? (Let the children respond with a "more" or "less" statement.) He reached down and picked up 8

charms. Then he decided to take 2 more and put them in his basket. (Use hand or cup) Now how many does he have in his basket? (10) He thought he would show his friend, Lulu has charms. On the way he stumbled and 3 fell out. Now how many are in his basket? While he was getting up, he thought the charms looked good enough to eat, so he ate 4 charms. Now how many does he have? (3) "Oh, boy! Look what I did," he said. "Lulu won't be excited about 3 charms!" He skipped back to his pile and picked up 7 more charms. Now how many does he have? (10) Off he went to see Lulu. When he reached Lulu's house in the tree, he had to climb to her door. The basket was too heavy. So, Loopy took out 5 of the charms and left them at the foot of the tree. How many did he take with him? How many did he leave at the foot of the tree? (5) Can we say the amount in the basket and in the pile by the tree are the same or different? He knocked at the door and when Lulu came he showed her his charms. She was excited and wanted some, too. So Loopy and Lulu climbed down to the pile and Loopy gave 6 to Lulu. Now how many does Loopy have? (4) How many does Lulu have? (6) Lulu didn't have a basket, so she gave hers to Loopy. How many are in the basket? They decided to go and play, so they put down their basket and went to play. While they were playing, Squeaky the mouse, came and ate 3. How many are in the basket now? (7) "Oh, those were delicious!", he said. So he ate 3 more. When Loopy and Lulu saw him eating their charms, they ran back. They looked in their basket and how many did they see? (4) They decided they would eat them before Squeaky had more. They both at the same amount. How many did they each eat? (2).

*EASTER:*

The Easter Bunny came hopping along the Easter trail with a beautiful basket in his paw. In his basket he had 4 red eggs and 4 yellow eggs. How many colored eggs did he have altogether? (8) He also had 7 polka dot eggs. 5 eggs had blue polka dots and the rest had pink polka dots. How many eggs had pink polka dots? (2) He hurried along until he came to his "secret basket tree," so he could get baskets for the children. He took out 2 baskets for the girls, and hung them on a branch. Then he went back and took out 3 baskets for the boys. How many did he take out? (5) On his way to the children's house with the 5 baskets, he tripped over a tulip, and 1 basket tumbled down the hill. How many baskets did he have then? (4) On his way back to the "secret basket tree", he met Mrs. Hen. She had 3 chicks with her. Then some more chicks hopped out from under a bush. Then there were 8 chicks. How many chicks came out from under the bush? (5) The Easter Bunny just happened to have 9 jelly beans in his coat pocket. He gave some to the chicks. Now he had 1 jelly bean. How many did he give to the chicks? (8) Mrs. Hen had something for Easter too. She had 3 chocolate eggs in her purse. The Easter Bunny had 5 chocolate eggs. How many more eggs did the

Easter Bunny have than Mrs. Hen? (2) Mrs. Hen, her chicks, and the Easter Bunny all sat down in the cool green grass and had a treat of chocolate eggs. Then the Easter Bunny waved goodbye and hopped way to deliver Easter joy.

## WORD PROBLEMS WITH NUMBERS

Introduction: When the fourteen lessons with the situations leading to addition and subtraction have been done often enough so that children can recognize when + and - need to be used in each case, start using numbers in these situations.

PROCEDURE: Children should have UNIFIX cubes to use to represent quantities in the problems.

Stage 1: Read the first problem slowly. After the children have used the UNIFIX cubes to find the answer, call on one child to give the answer orally. Ask the children whether they added the two numbers together or subtracted one from the other. Ask them to give a number sentence for you to write to show what is happening in the problem. Reinforce the meaning of the symbols. Proceed with other problems from the other lessons. Circle the number that answers the question in each case in each number sentence written.

**LESSON ONE (1)**
2 + 1 = (3)

**LESSON TWO (1)**
6 - 4 = (2)

**LESSON THREE (1)**
2 + (6) = 8

**LESSON FOUR (1)**
4 - (2) = 2

**LESSON FIVE (1)**
7 = (3) + 4

**LESSON SIX**
(9) - 2 = 7 OR 7 = (9) - 2

**LESSON SEVEN (1)**
6 = 3 = (9)

**LESSON EIGHT (1)**
9 = 3 + (6)

**LESSON NINE (1)**
**④ = 7 - 3 OR 7 - 3 = ④**

**LESSON TEN (1)**
**③ = 8 - 5 OR 8 - 5 = ③**

**LESSON ELEVEN (1)**
**⑦ - 3 = 4**

**LESSON TWELVE (1)**
**5 - ② = 3**

**LESSON THIRTEEN (1)**
**7 - 3 = ④**

**LESSON FOURTEEN (1)**
**⑥ - 4 = 2 OR**

Children should learn to always show the "difference" number in the comparison alone on one side of the number sentence. This provides (1) a focus on difference descriptions "more than", "less than", "fewer than", and (2 a consistent approach to the comparison.

Stage 2: Use the recording form supplied.

Read each problem slowly once after instructing children to concentrate on the question and finding the number that answers it. These should be entered on the recording form. Then read the problem slowly a second time after instructing children to think of the number sentence that shows what is in the language of the problem. This should be written in that space. Thirdly, have children circle the number in the number sentence that answers the question in the problem.

These number stories should be presented to children at least once each week. Below are some examples of number stories that can be used to parallel each of the fourteen situations. Many of these were developed by Nancy Berg and Carol Dyson, two primary teachers in Duluth, MN. You can develop similar stories of these 14 types to use. Mix these up so children get 3 or 4 different situations during each math period. Take time to discuss each. Show the correct number sentence on the overhead, with the correct number circled.

LESSON ONE:

1. "Kris brought two crayons to school. She found another one in her desk. How many pencils did Kris have?"

2. "During the first fall, the Pilgrms built 2 big log cabins. In the spring, they built 5 more log cabins. How many cabins did they build?"

3. "Donnie took 3 tree ornaments from the Christmas box. His mother gave him 4 more to hang on the tree. How many tree ornaments did Donnie have to hang?"

4. "Jack made 4 paper snowflakes. Tom gave him the 5 he had made. How many paper snowflakes did Jack have in all?"

5. "Jodie had 2 red candy hearts. Jane gave her 6 more candy hearts. How many candy hearts did Jodie have then?"

6. "On Mayday there were 7 kites flying in the sky. Then Fred's class sent their 2 kites up. How many kites were in the sky then?"

LESSON TWO:

1. "Charlie Chipmunk found 6 seeds. He gave 4 seeds to his mother to eat. How many did Charlie have to eat?"

2. "Tom has a box of 8 crayons. He gave 2 crayons to Sue to use. How many crayons does Tom have left in his box?"

3. "Indians who visited the Pilgrims had 5 bearskins. The gave 4 of these to the Pilgrims. How many did they have left?"

4. "Joyce found 7 toy soliders in a box. She gave her brother 4 to hang on the Christmas tree and saved the rest. How many toy soldiers did she save?"

5. "Pedro made 8 snowballs. He threw 5 at a can on the fence. How many did he have left?"

6. "Jill bought 7 valentines. She gave 6 to her classmates. How many did she have left?"

7. "Sam's kite needed more bows to fly. She had 9 bows on her kite's tail. She gave 5 of them to Sam. How many bows were left on her kite?"

LESSON THREE:

1. "2 school buses were parked by the school. Some more buses came and parked. Josie counted 8 school buses parked. How many buses parked late?"

2. "The Pilgrims have 3 sacks of corn to plant. The Indians gave them several more sacks of corn. The Pilgrims planted 6 bags. How many sacks of corn did the Indians give them?"

3. "Dan brought home 5 decorations made in school. Ann gave him those she had made in school when he got home. Dan had 8 decorations to put on the tree. How many decorations did Ann give him?"

4. "Tommy has 2 pairs of boots. His brother gave him those he had outgrown. Tommy now has 4 pairs of boots. How many pairs of boots did his brother give him?

5. "Jerry had 3 valentine suckers. Mack gave him some more. Jerry now has 7 valentine suckers. How many valentine suckers did Mack give him?"

6. "Gary's bows on his kite tail numbered 4. He added the bows Sue gave him. Now there are 9 bows on Gary's kite. How many bows did Sue give him?"

7. "Fannie's mother put 5 candles on the birthday cake. She saw she needed more so she put those on the cake. The right number of candles is 8. How many more candles did she put on the cake?"

LESSON FOUR:

1. "Sophie Squirrel gathered 4 acorns. She hid some under a log. She put the other 2 acorns in the nest. How many acorns were under the log?"

2. "The teacher had 6 erasers. She gave 1 eraser to each child in row 3 to use. She has 2 left. How many children are in row 3.

3. "Dan had 8 pieces of tinsel. He gave some to Ann. He has 2 pieces left. How many pieces did he give to Ann?"

4. "Bennie made 5 snowmen. His dog knocked some snowmen down. Two snowmen are still standing. How many snowmen were knocked down?"

5. "Sally has 8 valentines. She sent some to her friends. Now she has 2 valentines. How many valentines were sent to her friends?"

6. "Francie had 7 pieces of birthday cake. She put some in the cake saver. She gave the remaining 3 pieces to her friends. How many pieces were put in the cake saver?"

7. "Tom picked 9 dandelions on the way to school. He dropped some on the way. He gave 6 dandelions to his teacher. How many did Tom drop?"

LESSON FIVE:

1. "Chuck picked some hazel nuts. His brother gave him 4 more hazel nuts. Chuck had 7 hazelnuts to bring home. How many hazel nuts did Chuck pick?"

2. "Paul brought his marble sack to school. His friend Barnie gave him 3 more marbles. Paul now has 8 marbles. How many marbles were in his sack when he came to school?"

3. "John brought some bread loaves to the Indian village. They already had 4 loaves. The Indians shared 7 loaves of bread. How many did John bring?"

4. "Sally had some dried cranberries on a string. Terri put 3 more cranberries on it. There were 9 cranberries when they put it on the tree. How many cranberries did Sally have at the start?"

5. "Mike had some knit caps. His grandmother gave him 2 more. Now he has 5 knit caps to wear. How many did he have to begin with?"

6. "Kim had some valentines. She made 3 more in school and had 8 valentines to give to friends. How many did she have at the beginning?"

7. "Some kites were flying in the school yard. Tom sent 2 more into the air. Then there were 9 kites flying. How many kites were there at the beginning?"

8. "For her birthday Jane received some bows. She put these with

the 4 bows in her drawer. She has 8 bows now. How many bows did she get on her birthday?"

LESSON SIX:

1. "Susie Squirrel gathered some acorns. She dropped 2 on the way to her nest. She placed 7 in the nest. How many did she gather?"

2. "There was a stack of books on the class table. Mary took 3 of them. That left 2 for Fred to choose from. How many books were on the table at the start?"

3. "The Pilgrims put some wild turkeys in a pen. They took 3 to give to their Indian friends. Then there were 6 turkeys in the pen. How many turkeys did the Pilgrims put in the pen?"

4. "When John went to buy tree ornaments he saw a box with several in it. He bought 2 and counted 6 left in the box. How many were in the box to begin with?"

5. "Several snowflakes landed on Susie's mitten. She watched 5 melt and 3 were left. How many snowflakes landed on her mitten?"

6. "Some valentine candies were in a dish on the table. Steve took 2. When Jane came by she counted 5 candies in the dish. How many candies were in the dish to begin with?"

7. "Some kites were flying. After 3 were pulled down, 6 were still flying. How many were flying to begin with?"

8. "Some presents were in a pile on the table. When his mother gave Tom 2 of these, 5 were left in the pile. How many were in the pile at the start?"

LESSON SEVEN:

1. "Chuckie Chipmunk had 6 red berries and 3 green berries. How many berries did Chuckie have?"

2. "Jeanne has 3 reading books and 2 coloring books. How many books does Jeanne have?"

3. "When the Pilgrims and Indians had their first dinner together,

there were 4 pumpkin pies and 2 apple pies. How many pies did they have?"

4. "In making a wreath for the tree, Janet used 3 red loops and 5 green loops. How many loops did she use?"

5. "Kari has 2 red scarves, 3 white scarves, and 1 blue scarf. How many scarves does Kari have?"

6. "Tom had 2 heart valentines and 4 lace valentines. How many valentines did Tom have?"

7. "Gary found 2 fielder's gloves and 1 first baseman's glove. How many baseball gloves did Gary find?"

8. "Willie received 6 presents wrapped in green paper and 2 wrapped in white paper. How many presents did Willie receive?"

LESSON EIGHT:

1. "Bruno Bear found a bush with 9 berries. 3 were red and the rest were green. How many berries were green?"

2. "Hanging in the cloakroom are 8 jackets. 2 are red and the rest blue. How many jackets are blue?"

3. "On the Thanksgiving table were 5 bowls of vegetables. 3 had squash and the rest had corn. How many bowls had corn?"

4. "Ann had 8 Christmas tree ornaments. 6 of them had sparkles and the others had no sparkles. How many of the Christmas tree balls had no sparkles?"

5. "Tim has 6 pairs of mittens. 2 pairs are wet and the rest are dry. How many pairs of mittens are still dry?"

6. "Of the 7 jelly beans in a dish, 5 were red and the rest were black. How many jelly beans were black?"

7. "9 birds were sitting on the fence. 3 were blue and the rest brown. How many brown birds were on the fence?"

8. "All of Joyce's 7 presents were wrapped in colored foil. 2 were

in silver foil and the rest in green foil. How many presents were in green foil?"

LESSON NINE:

1. "Charlie Chipmunk saw 7 bears. Susie Squirrel saw only 3 bears. Charlie Chipmunk saw how many more bears than Susie?"

2. "Sue has 2 papers to bring home. Tom has 7 papers to bring home. Tom finished how many more papers than Sue?"

3. "For Thanksgiving, the Pilgrims had 6 turkeys and 3 fish. They had how many more turkeys than fish?"

4. "The Christmas tree light set had 8 blue lights and 4 red lights. How many more blue lights were there?"

5. "Jim made 3 snowballs and Fred made 6 snowballs. Fred made how many more snowballs than Jim?"

6. "Kelly received 6 valentines and Jolene received 4 valentines. Kelly received how many more valentines than Jolene?"

7. "When Jack and Sally went to pick flowers, Jack found 9 daisies and Sally found 4 buttercups. Jack found how many more flowers than Sally?"

8. "Jean and her twin sister Joan received birthday presents from their friends. Jean got 8 presents and Joan got 6. Jean got how many more presents than Joan?"

LESSON TEN:

1. "5 blackbirds were in one tree. 8 crows were in a second tree. How many fewer blackbirds were there"?

2. Bill brought 3 toy cars to school. Jim brought 6 toy cars to school. How many fewer did Bill bring?

3. All of the children went berry picking. The Pilgrim children picked 3 baskets of berries. The Indian children picked 5 baskets of berries. The Pilgrim children picked how many fewer baskets of berries?

4. The Christmas tree had 8 red lights and 6 blue lights. There

were how many fewer blue lights?

5. In their snow fort, Pat stored 3 snowballs. Betty had 6 snowballs stored. Pat had how many fewer snowballs in the fort?

6. When the teacher distributed the Valentine mail, Sam received 4 and Jean received 7. Sam received how many fewer valentines?

7. Sue had 9 fresh daisies and Sam had 6. Sam had how many fewer fresh daisies?

8. Tom had 4 candles on his birthday cake and his brother had 9 candles on his. Tom had how many fewer candles on his cake?

LESSON ELEVEN:

1. Charlie has 3 nuts in his pail. Susie has 4 more nuts than Charlie. Susie has how many nuts in her pail?

2. When the class went to the gym, the girls had 4 rubber balls. The boys had 2 more rubber balls. How many rubber balls did the boys have?

3. At the first Thanksgiving the children played games. The girls played 5 games. The boys played 3 more games than the girls played. The boys played how many games?

4. The Christmas tree had 3 presents with bows underneath. The presents without bows were 3 more in number than those with bows. How many presents were without bows?

5. Tom's team had 3 practice pucks. Sue's team had 4 more than Tom's team. How many hockey pucks did Sue's team have?

6. Joyce found 5 dandelions. Kristine found 3 dandelions more than Joyce. How many dandelions did Kristine find?

7. 4 school kites had red ribbon tails. The kits with blue ribbon tails were 3 more than this. How many kites had blue ribbon tails?

8. On her birthday, Tanya received 5 gifts. Janet received 2 more than this on her birthday. How many presents did Janet

receive?

LESSON TWELVE:

1. Charlie had 5 acorns in his pail. Susan had 3 fewer acorns in hers. How many acorns did Susan have?

2. Jim brought his set of 7 toy soldiers to class. Tom had 4 fewer soldiers in his set. How many were in Tom's set?

3. For the first Thanksgiving, the Indian children picked 6 baskets of berries. The Pilgrim children picked 2 fewer baskets. How many baskets did the Pilgrim children pick?

4. A Christmas tree had 9 red lights. There were 3 fewer blue lights. How many blue lights were on the tree?

5. Patty's snowman had 8 buttons. Jean's had 3 fewer buttons. How many buttons were on Jean's snowman?

6. Sam received 7 valentines. Sue received 2 fewer valentines. Sue received how many valentines?

7. Tammy found 5 dandelions. John found 1 dandelion less than Tammy. How many dandelions did John find?

8. Tom and Gail had birthdays two days apart. Tom's cake had 7 candles. Gail's cake has 2 candles less than Tom's. How many candles were on Gail's cake?

LESSON THIRTEEN:

1. Susie Squirrel has 7 animals in her family. She has 3 more animals than Charlie has in his family. How many animals are in Charlie's family?

2. Jean has 6 peanuts. She has 3 peanuts more than Pam. Pam has how many peanuts?

3. Indian children looped 7 poles. This was 3 poles more than the Pilgrim children looped. How many poles did the Pilgrim children loop?

4. Tom's chair had 9 presents on it. This was 3 presents more than Jane's chair. Jane had how many presents?

5. The Lost and Found Box in Room 10 had 8 pairs of mittens. This was 3 more pairs than Room 12 had. How many pairs of mittens were in Room 12's Lost and Found Box?

6. 8 valentines had flowers on them. This was 2 more than the valentines with hearts. How many valentines had hearts on them?

7. 8 kites flew on the sunny side of the school. This is 3 more than flew on the shady side. How many kites flew on the shady side?

8. Tom's 7 pencils for his birthday were 2 more than Jane's ribbons for her birthday. How many ribbons did Jane have?

LESSON FOURTEEN:

1. Joyce has 4 sunflower seeds. She has 2 fewer than Charlie. How many seeds does Charlie have?

2. Joe choose 3 library books. He chose 3 fewer than Deanna. How many library books did Deanna choose?

3. The Pilgrims had 5 ponies. They had 2 ponies fewer than the Indians. How many ponies did the Indians have?

4. Linda selected 5 of the ornaments. She selected 3 fewer than Tom. How many ornaments did Tom select?

5. Tony made 2 small snowmen. He made 3 fewer snowmen than Kathy. Kathy made how many snowmen?

6. Jake received 5 valentines. He received 3 fewer than Joannie. Joannie received how many valentines?

7. Bob's bag had 6 marbles. He had 3 fewer marbles than Gene. Gene had how many marbles?

8. Andy's 5 birthday presents were 2 fewer than his sister Gail's. Gail had how many birthday presents?

In creating and using these kinds of "story problems," use as many real objects as found in the problems as you can. Some suggestions are:

Beginning of school: Pencils, crayons, paper sheets, scissors, books notebooks and similar objects associated with this period of time.

Fall: Acorns and other nuts, leaves, seeds, and other things found on the ground at this time of year.

Winter: Snowmen, skates, skies, snowballs, mittens, tassel caps, jackets, overshoes and other things associated with this time of the year.

Spring: Kites, flowers, birds and other things related to this time.

Birthdays: Candles, presents, gifts, friends, etc.

Thanksgiving: Turkey cutouts, pumpkin cutouts, corn, etc.

Christmas: Unbreakable tree ornaments, gifts, tree lights, wreaths, bows.

Valentine's Day: Hearts, valentines, valentine candies

Easter: Jelly beans, eggs, paper rabbits, baskets

LEVEL ONE

MORE WORD PROBLEMS: Adding and Subtracting

1. Beth had 3 red balls. Jeff gave her 3 more red balls. How many red balls did she have then?

2. Ann had 8 flowers. She gave 5 of them to her mom. How many flowers did she have left?

3. Pam had 2 books. Sam gave her some more books. Then Pam had 5 books. How many books did Sam give her?

4. Jan had 10 sticks. She gave some to Sid and then had 4 sticks left. How many sticks did she give to Sid?

5. Bill had some cars. He gave 4 cars to Jack. Now he has 1 car left. How many cars did he have to start with?

6. Pete had some cans of pop. Jeff gave him 3 more cans of pop. Now he has 6 cans of pop. How many cans of pop did he have to start with?

7. Val has 5 green bows and 4 red bows. How many bows does Val have together?

8. Jake has 7 turtles. 5 are big and the rest are little. How many turtles are little?

9. Tom has 7 trucks and Sam has 6 trucks. How many more trucks does Tom have then Sam?

10. Becky has 7 rings. Jackie has 3 rings. How many fewer rings does Jackie have then Becky?

11. Ted has 7 kittens. Bill has 2 more than Ted. How many kittens does Bill have?

12. Todd has 9 pairs of socks. Ted has 2 fewer pairs of socks than Todd. How many pairs of socks does Ted have?

13. Sara has 4 marbles. She has 3 marbles more than Jean. How many marbles does Jean have?

14. Bob has 4 pencils. He has 4 fewer pencils than Jan. How many pencils does Jan have?

15. Rick has 1 rabbit. He has 4 fewer rabbits than Ron.
How many rabbits does Ron have?

16. Pam has 7 dollars. She has 2 dollars more than Jim.
How many dollars does Jim have?

17. Ron has 6 turtles. Sid has 4 fewer turtles than Ron.
How many turtles does Sid have?

18. Val had 1 dollhouse. Ann has 1 more dollhouse than Val.
How many dollhouses does Ann have?

19. Debbie has 5 birds. June has 9 birds.
How many fewer birds does Debbie have than June?

20. Joan has 4 pencils and Mike has 3 pencils.
How many more pencils does Joan have than Mike?

21. Betty has 9 rabbits. 6 are brown and the rest are white.
How many rabbits are white?

22. Mike has 6 trucks and 4 trains.
How many trucks and trains does Mike have altogether?

23. Gail had some beads. Jane gave her 6 more beads. Now she has 9 beads. How many beads did she have to start with?

24. Sam had some pigs. He gave 2 pigs to Frank. Now he has 7 pigs left. How many pigs did he have to start with?

25. Liz had 3 dogs. She gave some to Bob and then had 1 dog left. How many dogs did she give to Bob?

26. Kate had 1 sucker. Jill have her some more suckers.
Then Kate had 8 suckers. How many suckers did Jill give her?

27. Tom had 4 fish. He gave 4 of them to Bob.
How many fish did he have left?

28. Bob had 1 stick of gum. Val gave him 1 more stick of gum. How many sticks of gum did he have then?

29. Sam had 5 bugs. Tom have him 1 more bug. How many bugs did he have then?

30. Beth had 7 candy bars. She gave 4 of them to Ted. How many candy bars did she have left?

31. Jean had 3 balloons. Sue gave her some more balloons. Then she had 6 balloons. How many balloons did Sue give her?

32. Ted had 7 toy jets. He gave some to Ben and then had 5 toy jets left. How many toy jets did he give to Ben?

33. Tom had some hot dogs. He gave 2 hots dogs to Jan. Now he has 2 hots dogs left. How many hot dogs did he have to start with?

34. Pat had some marbles. Jill gave her 4 more marbles. Now she has 10 marbles. How many marbles did she have to start with?

35. Ron has 3 dogs and 2 cats. How many dogs and cats does Ron have altogether?

36. Jenny has 10 coins. 3 are dimes and the rest are nickels. How many coins are nickels?

37. Tina has 3 dolls and June has 9 dolls. How many more dolls does June have then Tina?

38. Mom has 8 oranges. Dad has 6 oranges. How many fewer oranges does Dad have then Mom?

39. Jan has 4 shells. Sam has 3 shells more than Jan. How many shells does Sam have?

40. Max has 6 fish. Jill has 5 fewer fish than Max. How many fish does Jill have?

41. Matt has 10 bugs. He has 1 bug more than Ben. How many bugs does Ben have?

42. Jack has 4 tops. He has 6 fewer tops than Tim.
How many tops does Tim have?

43. Liz has 3 oranges. She has 3 fewer oranges than Val.
How many oranges does Val have?

44. Joan has 8 necklaces. She has 2 necklaces more than Barb.
How many necklaces does Barb have?

45. Sue has 7 bananas. Rick has 1 fewer banana than Sue.
How many bananas does Rick have?

46. Jack has 6 ice cream cones. Liz has 4 ice cream cones more than Jack. How many ice cream cones does Liz have?

47. Rick has 2 dogs. Sue has 4 dogs.
How many fewer dogs does Rick have than Sue?

48. Mary has 10 blouses and Jill has 4 blouses.
How many more blouses does Mary have than Jill?

49. Ken has 9 apples. 2 apples are red and the rest are green.
How many apples are green?

50. Sara has 1 big duck and 6 little ducks.
How many ducks does Sara have altogether?

51. Todd had some dimes. Sue gave him 7 more dimes. Now he has 8 dimes. How many dimes did he have to start with?

52. Ann had some hats. She gave 1 hat to Ted. Now she has 9 hats left. How many hats did she have to start with?

53. Ann had 5 dresses. She gave some to Beth and then had 3 dresses left. How many dresses did she give to Beth?

54. Todd had 5 shells. Joan gave him some more shells. Then Todd had 9 shells. How many shells did Joan give him?

55. Rick had 10 rocks. He gave 3 of them to Pat.
How many rocks did he have left?

56. Jim had 4 apples. Jan gave him 3 more apples.
How many apples did he have then?

57. Pam had 5 shells. Liz gave her 4 more shells.
How many shells did she have then?

58. Joy had 5 dolls. She gave 2 of them to Val.
How many dolls did she have left?

59. Max had 7 ducks. Jim gave him some more ducks.
Then Max had 8 ducks. How many ducks did Jim give him?

60. Tim had 9 ships. He gave some to Val and then had 7 ships
left. How many ships did he give to Val?

61. Pam had some rings. She gave 5 rings to Liz. Now has has 4
rings left. How many rings did she have to start with?

62. Jack had some frogs. Tom gave him 1 more frog. Now he has
4 frogs. How many frogs did he have to start with?

63. Joy has 4 blue crayons and 4 yellow crayons.
How many crayons does she have altogether?

64. Matt has 6 balls. 4 are blue and the rest are green.
How many balls are green?

65. Ben has 2 packs of bubble gum and Beth has 5 packs of
bubble gum. How many more packs of bubble gum does Beth
have than Ben?

66. Jenny has 3 balloons. Jack has 10 balloons.
How many fewer balloons does Jenny have than Jack?

67. Tim has 5 trucks. Tom has 2 trucks more than Tim.
How many trucks does Tom have?

68. Kate has 10 stickers. Jenny has 3 fewer stickers than Kate.
How many stickers does Jenny have?

69. Fred has 9 Transformers. He has 3 Transformers more than
Chad. How many Transformers does Chad have?

70. Ann has 4 lollipops. She has 5 fewer lollipops than Pat.
How many lollipops does Pat have?

71. Beth has 5 apples. She has 2 fewer apples than Sam.
How many apples does Sam have?

72. Al has 5 kites. He has 2 kites more than June.
How many kites does June have?

73. Pat has 8 pairs of shoes. Joy has 4 fewer pairs of shoes than Pat. How many pairs of shoes does Joy have?

74. Pam has 3 skirts. Beth has 5 skirts more than Pam.
How many skirts does Beth have?

75. Jeff has 8 airplanes. Todd has 9 airplanes.
How many fewer airplanes does Jeff have than Todd?

76. Pete has 8 shirts and Tim has 5 shirts.
How many more shirts does Pete have than Tim?

77. Barb has 8 ice cream cones. 6 are chocolate and the rest are strawberry. How many ice cream cones are strawberry?

78. Jeff has 2 bats and 8 balls.
How many bats and balls does Jeff have altogether?

79. Kate had some pet rabbits. Joy gave her 5 more pet rabbits. Now she has 7 pet rabbits. How many pet rabbits did she have to start with?

80. Jill had some dolls. She gave 3 dolls to Sue. Now she has 5 dolls left. How many dolls did she have to start with?

81. Ron had 8 apples. He gave some to Joy and then had 6 apples left. How many apples did he give to Joy?

82. Chad had 4 guns. Tim gave him some more guns. Then Chad had 7 guns. How many guns did Tim give him?

83. Ben had 6 trucks. He gave 5 of them to Jack.
How many trucks did he have left?

84. Max had 2 blue cars. Tim have him 3 more blue cars.
How many blue cars did he have then?

LEVEL ONE

WORD PROBLEMS: More Than Two Numbers

Introduction: Introduce this extension to the children with a story. They should have UNIFIX cubes or other material to represent the materials. Ask them to use the materials and answer the question in the problem.

> "Tom has three balloons; Bill has four balloons and John has two balloons. How many balloons do they have together?"
>
> "Jean had three cookies and Patricia had two cookies. Jean gave one cookie to Theresa. How many cookies did Jean and Patricia have together?"
>
> "Tom had eight marbles. He had two more than John. John had two more than Bill. How many marbles did Bill have?"
>
> "Mr. Pirelli makes tires for bicycles and tricycles. He made enough for 4 bicycles and 3 tricycles. How many tires did he make?"

Devise as many problems involving adding and subtracting that show joining, separating, comparing and part-part-whole as you can.

Combine two of these situations into these problems as much as possible.

LEVEL ONE

PROBLEM CREATING

Introduction: Once children understand the origins of addition and subtraction in part-part whole, separating, joining and comparison situations; how to generate number sentences to represent these situations; and how to find numbers to answer questions, they are ready to create number "story problems."

Place three related numbers on the overhead, blackboard or flannel board:

3 8 5

Ask the children to think of a story that uses two of the numbers and asks a question for which the third number is the answer. Three different combinations can result.

| USE | ANSWER |
|---|---|
| 3,8 | 5 |
| 3,5 | 8 |
| 8,5 | 3 |

The children might think of any one of the fourteen situations involving part-part whole, joining, separating and "finding the difference" comparison.

Ask one child to orally give their problem. Write down what he/she says. Discuss the problem. What kind is it? How is the question asked? Does it use the given numbers correctly?

Ask each child to hand in the problem created. Read and check these. Categorize them as to type of problem. If any of the fourteen situations are not represented, you will have to spend time creating problems with the group and generate all of the types.

Use the children's problems as problems to read to children for them to manipulate materials and solve and create number sentences to represent.

Extension: Do this often enough with different number combinations so children become comfortable with writing "story problems" for addition and

subtraction. Don't fuss about incorrect spelling of words as long as the ideas are communicated. You can use these later in spelling lessons since they are words that are in the children's spoken vocabulary.

Later in the year, use 4 numbers with one being the answer, i.e.,

2 3 2 8

"John had 2 bags with 3 marbles in each bag. His brother gave him 2 more marbles. How many did he have then?"

LEVEL ONE

SYMBOLS: Checking on Symbol Phrases and Related Language

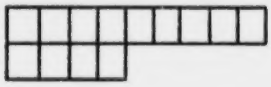
The children should have UNIFIX cubes to use. First make a link of 8 UNIFIX cubes of alternating colors so the children can see there are 8 cubes. Ask the children to make a UNIFIX link with 3 LESS THAN yours and hold it up for you to see.

Write 8 -3 on the board to show the number in their links.

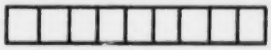
Then ask the children to make a link with 2 MORE THAN yours and show it to you. Write 8 + 2 on the board to show the number in their links.

Write several expressions using numerals and operation signs on the board. Ask the children to make arrangements of UNIFIX cubes to show what is indicated by each numeral phrase. Some examples:

9 - 4 show by:



or

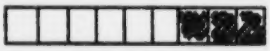


and

break off

leaving

or

6 + 3 shown by 

You will have to move around the room observing the children. For each one, urge them to find more than one way to show what the phrase means.

Symbol phrases and language to be worked with are:

| | | |
|---|---|---|
| more than | fewer than were left | |
| less than | altogether | were there then |
| which part | other part | together with |

PLACE VALUE: (Understanding Place Value)

Background: Place value understanding takes a long time to develop. There are several components to fully developed understanding of place value:

1. The group size used in grouping counted materials is arbitrary

2. Once a group size is chosen, it must be consistently used to further re-group smaller groups into larger groups

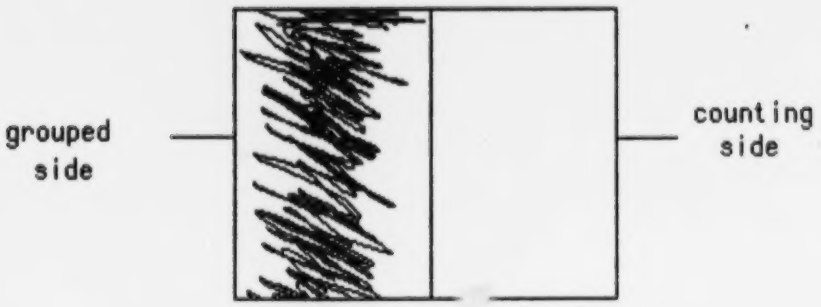
3. Once the agreed upon group size is reached, that group must be formed

4. Groups of larger size can be broken down to recover the smaller groups from which they are made.

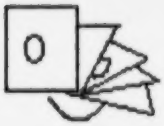
Since children are still developing numberness for numbers up through ten or slightly higher, it makes no sense to group by tens immediately. You will have them group by 3's, 4's, 5's, 6's, etc. in developing place value understanding.

The basic ideas in this development are to have the children count, group when the group size is reached, place that group to the LEFT of the counted, yet ungrouped materials. The sequence is (1) oral description of the materials on a place value mat, (2) gradual introduction of symbols to show the grouped and ungrouped quantities, (3) elimination of the oral descriptions and use of written symbols.

Materials used are a place value mat and countables such as UNIFIX cubes since they can be linked into the group size, numeral flips, recording sheets:




PLACE VALUE MAT




NUMERAL FLIPS with 0 - 9 on the cards to be flipped

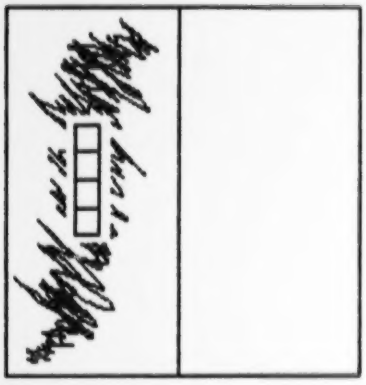
Activities are counting on and counting down.

LESSON ONE: Children should have the place value mats and UNIFIX cubes. Be sure they have the white side of their mats on the right.

Place a transparency place value mat like the children's on the overhead projector.

Put a counter on the clear side of the place value mat. Explain that this is the "counting side" and that the children will always put as many UNIFIX cubes on that side as you signal. Clap your hands once and indicate this is a signal to put one UNIFIX cube on the counting side. Tell the children they will keep putting cubes on the white side for as many times as you signal and until the 'MAGIC NUMBER' IS REACHED. When the MAGIC NUMBER is reached, the children will link those cubes and put them on the colored side.

Demonstrate on the overhead "This MAGIC NUMBER is four". Slowly add cubes one at a time until 4 are on the white side. Link these and put them on the colored side:



Ask the children how many loose cubes could be on the white side if the MAGIC NUMBER is five. "What would you do if you had five cubes on that side?" Talk about needing to group when the MAGIC NUMBER is reached. Explain to the children that the numbers will each have their own name. You name a group of three as a BOSCO. Ask them to agree on names for groups of four, five, six, seven, eight and nine. Tell the children you will play a counting game with them often with all of the named groups.

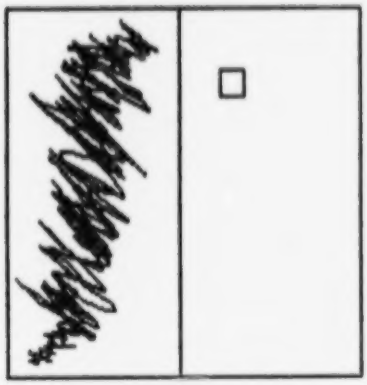
LESSON TWO: Children should have place value mats and UNIFIX cubes.

Introduction: Tell them the magic number is the name chosen for FIVE by the children.

> "We are going to use the UNIFIX cubes to make a counting chart in "_____" land. The symbol will be a hand clap. With each hand clap you hear place ONE Unifix cube on the white side of the mat. Remember that when the magic number "_____" is reached, you MUST group the cubes and put them on the colored side. At each step, when I ask you, tell me how many "_____"s and how many ONES you have on your mat. Ready?"

(CLAP)

Children should have:



"What is on your mat? - all together". Response should be "zero '_____'s and ONE."

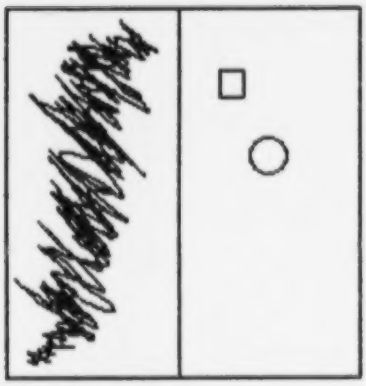
On a two column form on the blackboard, write:

| | |
|---|---|
| 0 | 1 |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

Point out to the children this shows 0 "_____"s - point to the 0 - and 1 - ONE - point to the 1.

(Clap)

Children should have:



"What is on your mat? - all together." Response should be "zero '_____'s and TWO." Write:

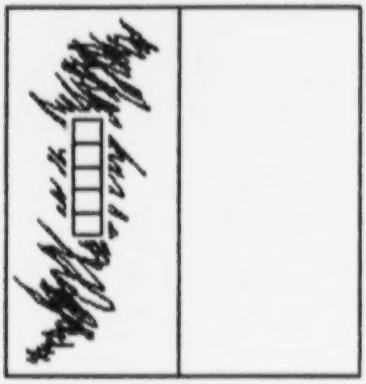
| | |
|---|---|
| | |
| 0 | 1 |
| 0 | 2 |
| | |
| | |
| | |
| | |
| | |
| | |

Ask the children what the "2" shows and what the "0" shows. Clarify any misconceptions.

Continue with one clap at a time until there are five UNIFIX cubes on the counting side. Pause long enough for children to realize they must group there and put the link on the other side. If necessary ask the children to tell you what must be done when the magic number of UNIFIX cubes is on the counting side. Reinforce this idea and make certain ALL children have made the link and put it on the group side.

"Tell me what is on your mat - all together." Response should be "ONE '_____' and ZERO."

MAT



CHART

| | |
|---|---|
| 0 | 1 |
| 0 | 2 |
| 0 | 3 |
| 0 | 4 |
| 1 | 0 |
| | |
| | |
| | |

Continue counting on and forming groups. At each stage the children must orally describe what is on the mat and you must write on the counting chart. When the magic number has been reached for BOTH sides, i.e. 44. Ask the children what can be done. Some may suggest grouping the ONES into a FIVE and putting that on the other side. Then ask if it is legal to have the magic number there and what can be done. Someone is likely to suggest another column is needed to put FIVE FIVES into a group. If not, suggest it and point out the two column form is now full and no more counting can done in "_____" land.

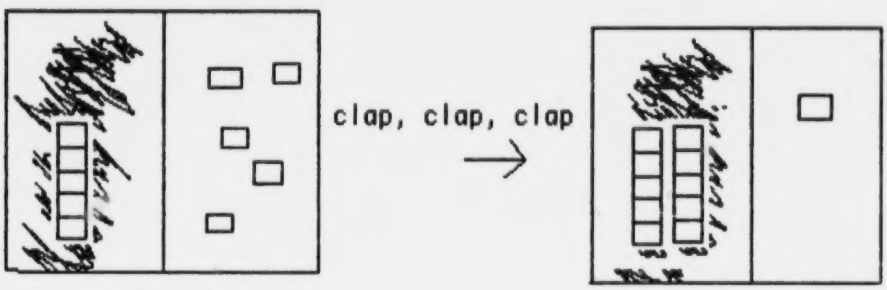
Extensions: Play this counting on game with the children to develop full 2 column counting charts for three, four, five, six, seven, eight, and nine. Prepare large models of these and hang them up in the rooms:

| BOZOS | ONES |
|---|---|
| 0 | 1 |
| 0 | 2 |
| 1 | 0 |
| 1 | 1 |
| 1 | 2 |
| 2 | 0 |

etc.

At a UNIFIX number station, place blank two column forms and ask the children to go to the number station, count UNIFIX cubes one at a time and develop their own counting charts.

Play the counting game with oral descriptions only by having the children add the cubes to the counting side in varying amounts, ie., 1,2,1,1,3,1,2. Judge these amounts by the grouping side used. Have them add so that a group is formed using only some:




Start with a given amount and then count on from that amount, ie., "one Bozo and two" to start.

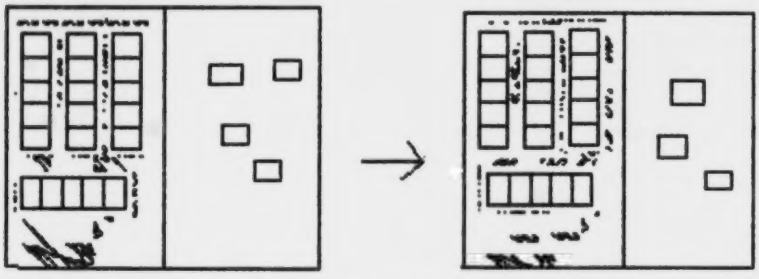
LEVEL ONE

PLACE VALUE: (Counting Back)

Background: When children recognize what is occuring in FORMING groups, they are ready to begin DECOMPOSING groups. This is done by playing a counting back game.

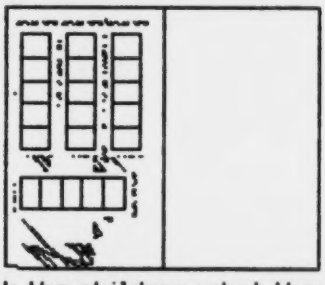
Introduction: Have the children start with a "full" place value mat for the given Magic Number, ie. 44 for a magic number "5".

Introduce a second signal - a tap of a ruler, a snap of the fingers, the ringing of a bell, etc. to show counting back or taking counters off the board. Emphasize that is done by removing from the COUNTING side only for now. (SNAP)



"What is on your mat? - all together" Response: "Four '_____' and Three."

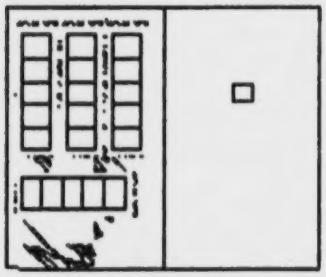
Continue having them remove the cubes ONE at a time until:



is reached

Ask the children what they would do if you gave a signal to take one cube away. Discuss the response and make it clear that ONE of the groups would have to be broken apart and the cubes from it put on this counter side. Only then could one cube be removed. Continue with the game.

When the board reaches:



ask the children what would happen if you snapped TWICE. Discuss these responses and reinforce the idea that when not enough cubes are on the counting side to take off what is required, a group must be broken down and its counters put on that side.

Continue on until the board is "clean."

The counting back game should follow the same sequence:

oral representation
oral + numeral flip representation
numeral flip representation
numeral flip and written record representation
written record representation

The counting back game should be played with ALL grouping sizes and by varying the number of cubes indicated by the signal.

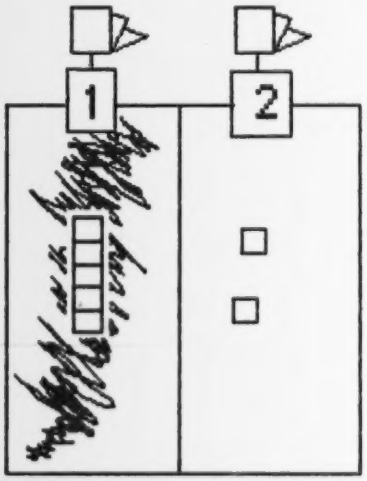
Extension: When children can easily count onto and count off the place value mat by the above games, integrate them by mixing the signals for adding on and taking off. This should be done when the children are recording in the 2 column forms only.

LEVEL ONE

PLACE VALUE : (Introducing Numerals)

Background: Once children count on and form the groups as needed well, and give accurate oral descriptions in terms of groups, you can introduce the numeral flips - one for each column

Introduction: Tell the children you are going to play the counting game with one added part. On the overhead projector version of the place value mat, show them how the numeral cards are flipped down into each column to show how many of what is supposed to be in that column is there. You can use:




Numeral flips made of transparency material shows this.
Introduce the numeral flips to the children. Have them arrange one above each column on the place value mat. Walk around to make sure each child has them arranged so the numerals flip over and show in the sequence: 0, 1, 2, 3, etc. When satisfied everything is set, choose a magic number and ask the children to make "three '_____' and two" on the mat and flip the numerals to show this. Be sure ALL can correctly represent a quantity consisting of part grouped and part counters correctly. Repeat with other amounts if needed until they can correctly do this.

Activity: Continue with the counting on game as before. At each step, have the children (1) orally describe what is on the mat and (2) flip the numerals to show what is on the mat.

Extensions: Do this frequently with different group sizes having both oral and numeral representations at each stage.

LEVEL ONE

PLACE VALUE: (Focus on Numerals)

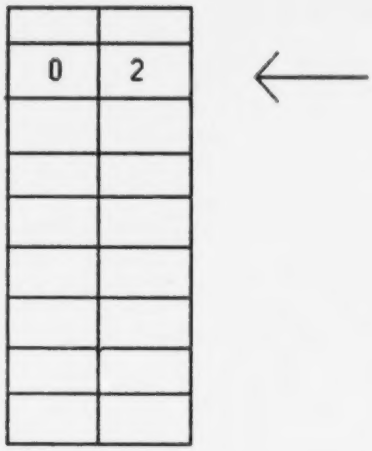
Background: At this stage of the sequence, the oral descriptions are discontinued and written symbols are coordinated with the numeral flips. This requires the use of 2 column recording forms by each student. When playing the counting on game, the results at each stage are represented by (1) arranging the numeral flips and (2) writing the numerals on the flip cards showing onto the 2 column recording forms.

Introduction: Each child is given a 2 column recording form to use in addition to the place value mats, UNIFIX cubes and numeral flips.

Tell the children they are no longer to describe the place value mat arrangements orally.

Start the game, at the first stage, after the children have flipped the numeral cards, tell them to write what is on the card for each column in the first row of the 2 column recording form. Put the correct representation on the blackboard or the overhead projector so they can verify that what they have written is correct.

Example:



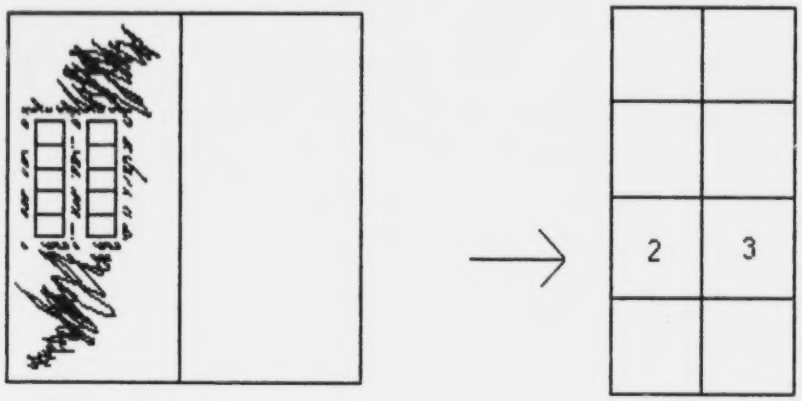

Remind the children that from now on the game will be played by (1) flipping the numeral cards, and (2) writing numerals on the recording forms.

LEVEL ONE

PLACE VALUE: (Recording Only)

Background: When children can accurately transfer from the numeral flips to the recording form a representation for what is on the place value mat, eliminate the use of the numeral flips and have them directly record the results of activity on the place value mat onto the recording forms.

Example:



LEVEL ONE

PLACE VALUE : (Last Words)

If the children develop number concept so that several of them master ten, you could use ten as the magic number in the counting games. This group size should be given the name TEN.

Play all of the counting games using TEN. The UNIFIX cubes are best for this work with TENS.

Alternatives to Unifix Cubes: The children can use BEANS as counters and small CUPS to hold the Magic Number of beans when reached in the counting games.

The children can use WHITE Cuisenaire rods as counters and TRADE for light green rods, purple rods, etc., as the Magic Numbers of 3 and 4, etc. are reached.

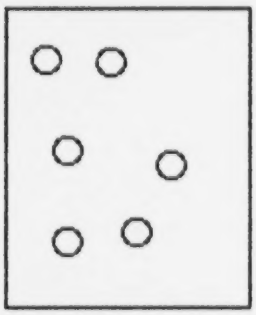
LEVEL ONE

NUMBER OPERATIONS : (Multiplication and Division)

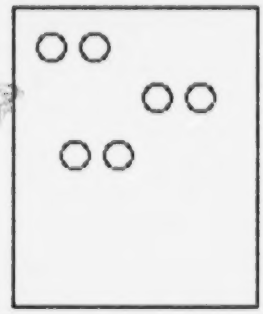
Background: In assessing number concept develop using the handtest, questions like "How many two's are in my other hand?" and "How many threes are in my other hand?" should be asked when determining if a child has developed "sixness". This should be done for all even numbers and multiples of three. When several children can successfully answer questions like these, increased emphasis should be placed on these operations.

LESSON ONE:

Introduction: On the overhead, place six (6) chips:



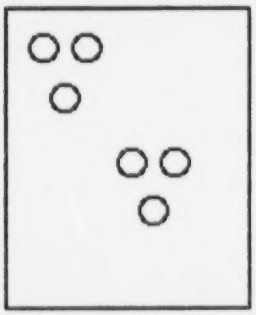
Arrange these in two's:



"How many two's did the six make?"
"How many two's are in six?"
"Three of what make six?"

Write 3(2) = 6, explaining you are putting the collection size in ( )and the 3 to show there are 3 two's. Follow up by asking how many two's are in four. Show this as: 4 = 2(2)

Rearrange the chips so that there are three's:



"How many three's did the six make?"
"How many three's are in six?"
"Two of what make six?"

Write this as: 6 = 2(3) and point out two new ways to write 6:
6 - 2(3) and 6 = 3(2)

Activity: Give each child several UNIFIX cubes or beans. Tell them to take four and make these into two's - link the UNIFIX or group the beans. Ask them to write the number sentence to show this on the lap board. Be sure all can write 4 = 2(2) to show how many two's in four. Have them add one more so they have five. "How many two's can be made from five?" "What of the one left over?"

Write 5 = 2(2) + 1 to show five gives 2 two's + one more.

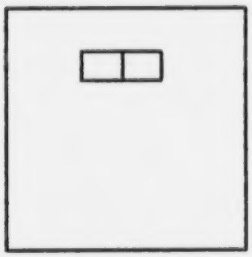
Ask the children to find how many two's are in each number through twelve and have them write number sentences to show this.

Follow up by having them make threes of each of these numbers. Ask questions like: "Does ten give more two's or more threes?"

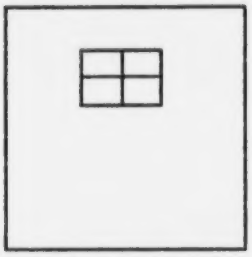
Do this for numbers as large as the children can count.

LESSON TWO:

Introduction: Using chips or squares on the overhead, repeatedly add to an array of two's. First put one two:

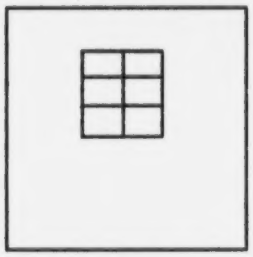


Put a second two as shown:



Ask the chilren how many two's are there. "How many chips (or squares) are there?" Write: 2(2) = 4.

Place another two into the array:



Ask the same questions, repeating or elaborating as necessary and finally, write: 6 = 3(2).

Activity: Children should have UNIFIX cubes or tiles. Ask them to keep adding rows of two to an array, one at a time and write a number sentence each time. They are limited only by the numbers that they can write. Numbers larger than ten will be accessible to those who have internalized place value concepts.

Extensions: Have them put together rows with three, then four, then five as they are able, writing number sentences each time.

LEVEL ONE

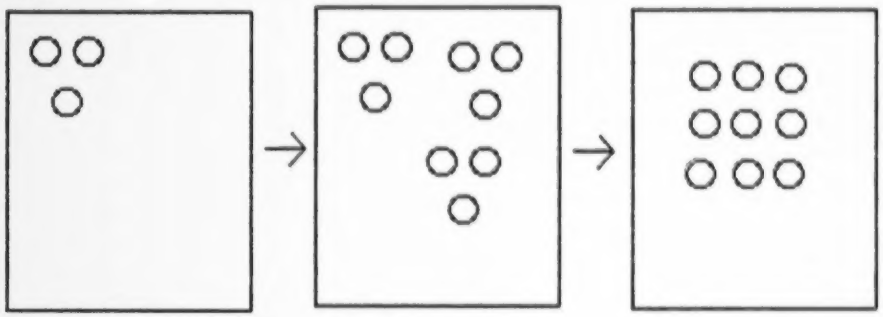
WORD PROBLEMS: Multiplication - No Symbols

Background: After children have had much experience with comparing groups of things, joining groups of things, separating larger groups of things into smaller groups in different ways, they can be given word problems orally that require them to use multiplication and division - at first without symbolizing these operations.

Introduction: Children should have some countables to use to represent things mentioned in the problems. They are to manipulate the materials in response to the action indicated in the problem.
Present a problem:

"Tommy put 3 pennies in his bank. Show Tommy's 3 pennies. He put this many pennies in the bank for 3 days. How many pennies were in the bank?"

On the overhead, place a group of 3, then two more as shown and indicate to the children the three groups of 3 that represent ALL of the pennies in the bank:



"So he had 9 pennies in his bank."

Give another problem:"Sammy Squirrel found 4 bushes that each had 2 berries. How many berries did Sammy find?"

Repeat the overhead process of representing each bush and the total number of bushes if needed.

Give as many additional problems as needed to get children to use this repeated addition interpretation of multiplication successfully. Encourage the children to array the materials representing things in rows:

Berries on bushes:

berries - how many bushes?

LEVEL ONE

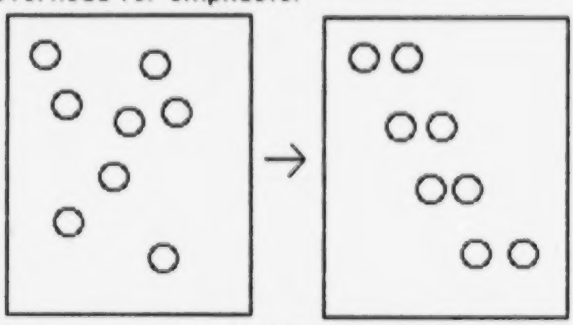
WORD PROBLEMS: Division (Measurement) - No Symbols

Introduction: Children should have UNIFIX cubes, beans or other countables to represent things in the problems. They should manipulate the materials to show the action of the problems.

Give them a story:

"Bunny Bob had 8 easter eggs. Show his 8 easter eggs. He gave 2 eggs to each child in the family. How many children are in the family?"

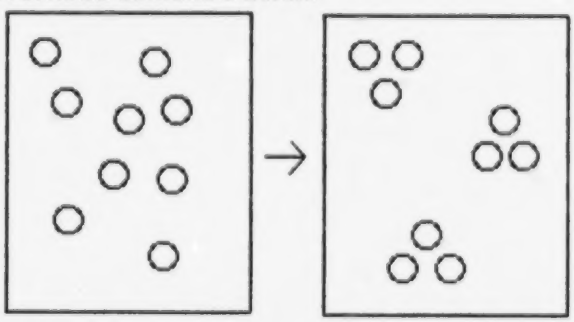
After they have separated the 8 things into two's, demonstrate this on the overhead for emphasis:



"Two eggs for each child so there are 4 children." Point to each of the four and count them. Give a second story involving this measurement or repeated subtraction kind of divison.

"Frances had 9 cookies. She put 3 cookies in each bag. How many bags did she use?"

Again, observe how the children break up the 9 things and repeat the overhead demonstration.



Count the three groups of 3 so "there are 3 bags used."

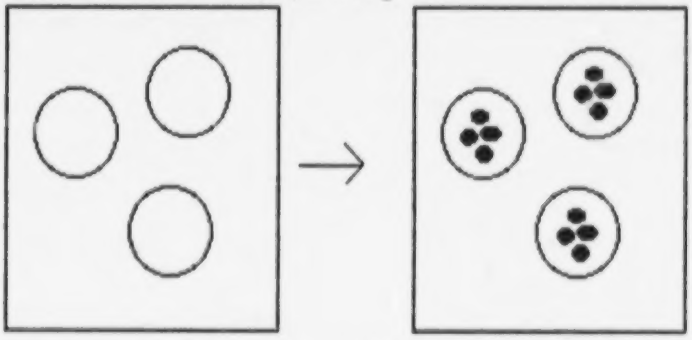
Activity: Give additional division stories of this kind until the children are comfortable manipulating the materials and finding the answers by dividing a large group into smaller groups of a given size and counting the number of groups formed.

LEVEL ONE

WORD PROBLEMS: Division (Partition) - No Symbols

Introduction: Children should have beans and small cups. Give the children a problem: "Joanie has twelve crayons. Show the twelve crayons with beans. She wants to give the same number to each of three classmates. Show the classmates by cups. How many crayons did each classmate get? Put beans in the cups so there are the same number in each cup. How many beans are in each cup?"

Observe what the children have done. Repeat this on the overhead for emphasis. Use a transparency with three circles:

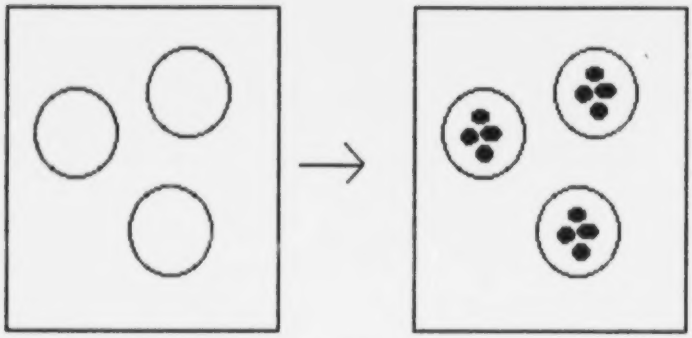


Show the children twelve beans. Put one in each circle until all are distributed, and point out that there are FOUR in each circle.

Follow with a second story. Remind the children to use beans and cups to work the problem.

"Peter has eight marbles to put into four bags. How many marbles are in each bag?"

Watch to see the children are using 4 cups to show the 4 bags and EQUALLY distributing the beans in these. Again, emphasize the re-enacting of the problem on the overhead. Use a 4 circle transparency:



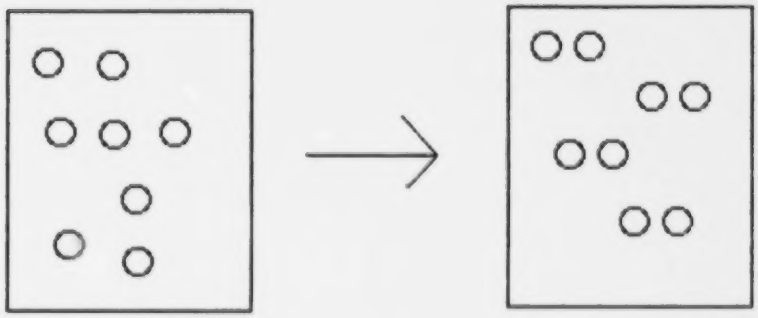
"There are TWO marbles in each bag." Do as many of these partitioning problems as needed to get children to adopt a process to ensure AS MANY IN EACH of the groups specified.

LEVEL ONE

NUMBER OPERATIONS: Signs for Multiplication and Division

Background: Children will not work extensively with multiplication and divison until the second level, but they should learn how the operation signs are used for the simplest joining and separating cases.

Introduction: On the overhead projector place eight counters. Tell the children you are going to divide these into groups of two and do that so they can see it taking place:



Point out that when we find how many two's are in eight we write:

8 ÷ 2

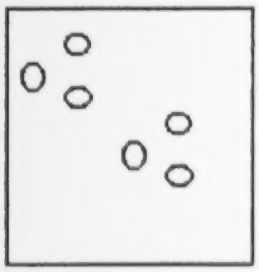
"How many two's are on the overhead?"

Complete the number sentence as:

8 ÷ 2 = 4

to show there are four 2's in 8.

Put six chips on the overhead. Ask the children how many three's can be made. Make them:



Write $6 \div 3 = 2$

and emphasize 6 is what we have at the start; 3 is the size of the collections we make; 2 is the number of these collections.

Write

$8 \div 2 = 4$

and ask the children to tell you what 8 represents, 2 represents and 4 represents.

Activity: Have the children make links of four Unifix cubes and break off two links. Have them write the number sentence to show this on the lapboard. It should be:

$4 \div 2 = 2$

Re-explain or elaborate as needed. Have them see how many two's they can make from several numbers you assign and ask for the number sentences. Assign one odd number and ask the children what should be done with the remainder when this results. You might suggest:

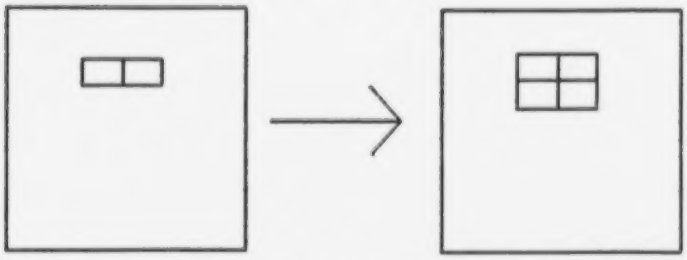
$7 \div 2 = 3 \text{ R } 1$

as a way to show this.

Extensions: Repeat this activity with other objects being grouped into smaller collections, beans being put 2 or 3 in each of several cups, etc. Have the children write the number sentences to show this.

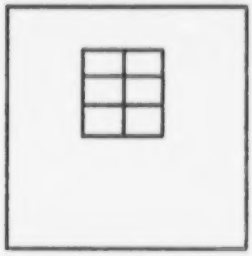
LESSON TWO

Introduction: Place 2 rows of 2, one at a time into an array:



Ask the children how many two's there are. "How many tiles are there?" "To show several of the same thing we use 'x'."

Write: 4 = 2 x 2. 4 shows the total, the second 2 shows the number in a collection; and the first 2 shows how many of the collection there are. Add another row of two:



"How many tiles altogether?" Write 6
"How many are in each row?" Write 6 = 2
"How many rows are there?" Write 6 = 3 x 2

Remind them that 6 is the total, 3 shows how many equal things, and 2 shows the size of each.

Activity: Each child should have UNIFIX cubes or tiles to use. Have them make successively greater numbers of rows of two, writing the number sentence each time.

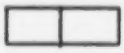
1 x 2 = 2 and 2 = 1 x 2

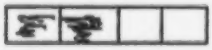
2 x 2 = 4 and 4 = 2 x 2

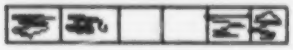
3 x 2 = 6 and 6 = 3 x 2

When you are sure they are writing these correctly, introduce the recording forms and have them enter the number sentences in these.

Extensions: Gradually increase the size of the groups being joined to three, four, etc. Use tiles and array. Use UNIFIX links and add together to get longer links.

 1 x 2 = 2

 2 x 2 = 4

 3 x 2 = 6, etc.

LEVEL ONE

WORD PROBLEMS:Multiplication and Division - No Numbers

Background: After children have had extensive experience in joining and separating and comparing groups equal in size or multiples of others, they can be introduced to word problems that involve multiplication and division. This should be done without numbers first so they learn to interpret the language that suggests these operations.

LESSON ONE

Introduction: Children should have counters of some kind to use to represent objects in the problems.

"Carol, Sue and Felix each have the same number of crayons. How many pencils do all three have?" What do you do to find the answer? The children may respond with "addition" or "multiply."

Use materials on the overhead with the transparency and give several examples of different amounts that each have. Show how the adddition can be done by multiplication since all of the parts are equal.

| Carol | Sue | Felix | |
|---|---|---|---|
| o o o | o o o | o o o | 3 + 3 + 3 or 3(3) or 3 x 3 |
| o o | o o | o o | 2 + 2 + 2 or 3(2) or 3 x 2 |
| o o o o o | o o o o o | o o o o o | 5 + 5 + 5 or 3(5) or 3 x 5 |

"Joannie bought several candy bars. Each bar cost the same amount. How much did she pay for all of the candy bars? Discuss this problem, again pointing out the parts are all equal (cost the same.)

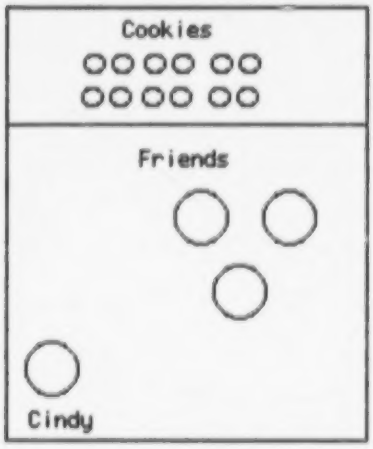
"The first grade class collected the same number of soup can labels each day for several days. How many soup can labels did they collect?" Discuss this. Represent the soup can labels with objects and array rows of these in whatever number the children suggest as being the days.

LESSON TWO

Introduction: Read the following story:

"Cindy had some cookies to share with some of her friends. How many cookies did each get?"

Discuss this. Have the children suggest (1) the number of cookies to be shared and then (2) the number of friends. Use the overhead to separate objects into the groups using quantities suggested by the children.




Use a transparency. Draw circles on it to indicate the number of friends suggested by the children.

Example:

12 cookies 3 friends

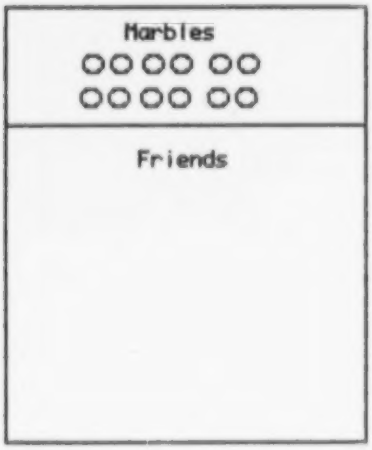
Discuss "equal sharing" and the need to have each friend get just as many cookies. Refer to "dividing" the cookies up, and that the number of cookies is divided into smaller groups. Does Cindy get some cookies, too? Follow with another similar story until the children have experience with developing a process that assures equality of the groups being formed.

LESSON THREE

Introduction: Read the following story:

"Tom has a can of marbles. He gave a certain number of these to several friends. How many friends received marbles?"

Again, use the overhead projector and sort materials into collections, using the numbers suggested by the children.




"Two to each." Does Tom get to keep some marbles when they are "shared?"

Extensions: Repeat these lessons often during the later part of the year. The objective is to give children experience with the "repeated addition" idea of multiplication, the "repeated subtraction" or measurement interpretation of division and the "sharing" or partitive interpretation of division.

LEVEL ONE

MULTIPLICATION AND DIVISION PROBLEMS

A. Students should have manipulative materials to use: tiles, beans and cups.

B. Students should record the answer on the recording form after the first reading.

C. Students should write a number sentence for the problem after the second reading.

---

1. Angela's mother had eight yards of rope. She cut this into four equal pieces for ten ropes. How long is each piece of rope?

2. There are four hamburger buns in a bag. How many bags should you buy to have buns for eight hamburgers?

3. Renee had three tanks of goldfish. There were three fish in each tank. How many goldfish did Renee have?

4. Maria found four bags of marbles. Each bag had two marbles. How many marbles did Maria find?

5. Greta practiced piano for ten hours last week. She practiced the same amount of time each day for five days. How many hours did she practice each day?

6. Robert had nine extra crayons. He gave the same number to Bob, Tom and Joan. How many crayons did each child get?

7. Jack found he could make links with two UNIFIX cubes without any left over. He made five links. How many UNIFIX cubes did he start with?

8. Fred's sister brought eight cans of pop to the party. Each child got two cans of pop. How many children received cans of pop?

9. Tom sold four baseball cards each to Tim and Mike. How many cards did he sell?

10. Fred needed two new bolts on each wheel of his wagon. How many bolts did he need?

11. Three children each found three acorns on a science walk. How many acorns were found?

12. Grace put eight beans in four cups so there were the same number of beans in each cup. How many beans were in each cup?

13. Ten children put the shoes they were wearing in a pile. How many shoes are in the pile?

14. Twelve sweet rolls will serve how many children if each receives three?

15. Fifteen bottles will fill how many cartons that hold five bottles each?

PROBLEM SOLVING: ASKING QUESTIONS

Background: One indication of children's ability to apply arithmetic operations to problems is being able to supply questions that relate to given information.

Introduction: Read a problem.

> "Iris had three gifts at Christmas and two on her birthday."

Ask the children for a question to ask about Noreen's gifts. Follow each proposed question by asking what operation would be done on the given numbers to answer that question. Discuss fully.

Children should have countable materials to represent the objects in the problems. More problems to use:

1. Debbi had three bags with three things in each bag.

2. Richard had six chairs. He put them at three tables. He put the same number of chairs at each table.

3. Greg had 9 toy cars. He gave 6 to Gary.

4. Harvey has 7 marbles. Vicki has 4 fewer marbles than Harvey.

5. Kristy had six cans of pop. Two friends came to visit her. Each one drank the same number of cans of pop.

6. Two cars had four tires on each car.

7. Susan Elizabeth had three unifix and went and got six more.

8. There were five boys in the first grade. Some of them were sick.

9. Mrs. Irons had eighteen students in the class. She divided the class into six equal groups.

10. Emma had six boxes. She put them on the rug in two stacks.

She put the same number in each stack.

11. There were 6 kittens. 3 were covered with a blanket.

12. Betsy had 9 multilinks. Six multilinks are red. The others are blue.

13. There were 12 steps from the basement to the upstairs. Erin took two steps at a time.

14. Two things weigh four grams each.

15. Three cartons of milk were empty and four were full.

16. Mark had 2 marbles. Mary had two more marbles than Mark.

17. Robert had three friends who shared eight of his cookies equally.

18. June had six cans. She put them in two different cupboards. She put the same number in each cupboard.

19. Edith had 4 dolls. She would like to have 7 dolls altogether.

20. Steve had some balloons. After the teacher gave him 6 more, Steve had 9 balloons.

21. Brian and six of his friends shared fourteen cookies. Each one got the same number of cookies.

22. There were four groups of five students.

23. Wendy baked two cakes. Each cake had three layers.

24. Four students were sitting around a table. One more student joined them.

25. Mark had three multilinks. Dick gave him four more.

26. Ember fixed three plates of spaghetti. Each plate had four meatballs on it.

27. Bob had six books. He put them into three piles. Each pile had the same number of books.

28. Kelley had 8 candy bars. Three of the candy bars were in her jacket pocket. The rest were in her hand.

29. Greg had 9 multilinks Greg gave some to Shelley. Then Greg had 6 left.

30. Carrie Lynn had seven toys. She put them on three tables.

31. Linda had three candy bars. She had three friends that wanted a candy bar.

32. Mrs. Weir went shopping and bought three cans of peas, two cans of tomato soup and one can of coffee.

33. Shirley had six boxes. She put two boxes on the table and the rest of the boxes on the floor.

34. Frank fixed three plates of biscuits with four biscuits on each plate.

35. Ten students made two lines. There were the same number of students in each line.

36. Tom had 6 marbles. His dad gave him some more marbles. Tom now had 10 marbles.

37. Pat had 8 paper dolls. Laura has 6 paper dolls.

38. Mrs. Jordan had twenty four multilinks. There were twelve students in the class. Each student got the same number of unifix.

39. Larry went to the store and bought two pieces of candy. Each piece of candy cost ten cents.

40. There were 11 students in the first grade. Mrs. Durst made 3 groups.

41. Al and four of his friends were playing football on the field. Three of their friends were off the field watching them.

42. Three boys stood by the window. Two girls stood by the door.

43. Trudy had six pairs of shoes.

44. Lynn had five candy bars. She gave four of her friends the same number of candy bars.

45. There were five chairs in each row. There were four rows of chairs.

46. Nancy bought three apples and two bananas.

47. Derek had three winter coats. Each coat had six buttons.

48. Amanda and Kelly joined six of their friends and went out to play.

49. There were five sets of beans. Each set had four beans.

50. Shelly and five friends were playing in her room. Four were jumping on the bed and some were waiting their turn.

51. Dean built five things out of multilinks. Each thing had six multilinks.

52. Vicki ate six cookies. Karen ate two more than Vicki.

53. There were six boys and seven girls in the line waiting for lunch.

LEVEL ONE

ARITHMETIC OPERATIONS: WORDS

Background: The children need to use, and think about, language associated with the arithmetic operations. Take the time for discussion.

Introduction (using addition): Say "add". Ask the class to tell you what they think of when they hear that word. Discuss the responses with the class. Other words to use:

| | | |
|---|---|---|
| subtract | plus | equal |
| multiply | minus | remainder |
| divide | share | |

Some phrases to use:

the difference between
more than
less than
times as many
equal

Symbols to use for interpretation and discussion:

+, -, =, x, , all numerals

LEVEL ONE

PROBLEM SOLVING: STRATEGIES

Introduction: Problem Solving is more than finding numbers to answer questions in number "stories." Many problems don't involve numbers. Some problems have no answers. Some problems have many answers. Children must know how to approach problems of all kinds, and some specific techniques that are helpful in solving problems. Some of these that children of this age are capable of handling are:

Guess and check
Draw a picture
Look for patterns
Make a table or list
THINK

Strategy Activities

Activity 1:
Example: Look at all numbers in a problem.

"Charles has a red can with 4 marbles, a blue can with 5 bottle caps and a green can with 4 walnuts."

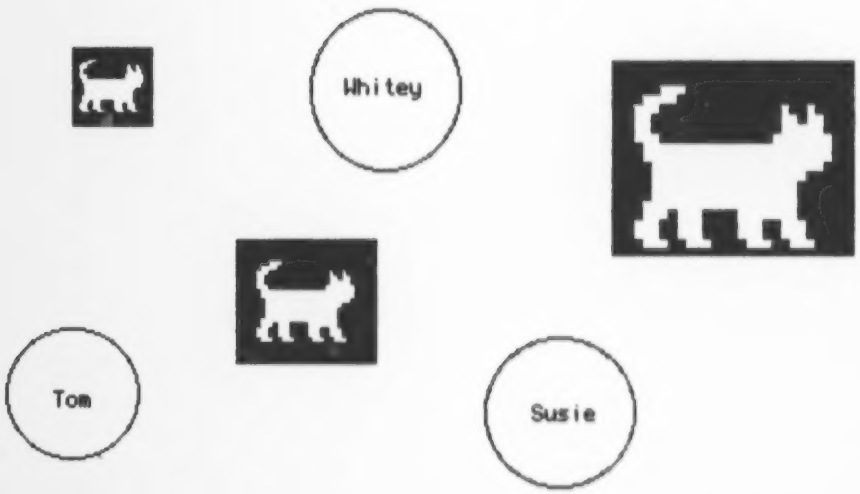
"How may cans does Charles have?"

"How many things does Charles have in his cans?"

"Which color can has the most things?

Activity 2:
Have children draw lines to show relationships and pictures to help their thinking.

Example: "Celia had 3 cats. The largest was brown and named Tom. The middle size cat was striped and named Susie. The smallest was white and named Whitey. Draw lines and color the cats to show this:




Activity 3: Have children make tables. "Mints are 2 for 5 cents. Make a table to show how much 4,6,8 and 10 mints would cost."

| number of mints | 2 | | 6 | |
|---|---|---|---|---|
| cost | 5 cents | 10 cents | | 20 cents |

Activity 4: A parade of loons crossed the lake. Two loons were in front of the biggest loon and thre were behind it. How many loons were on the lake?

Activity 5: Have children choose which nunmbers to use in problems:

"Bobby went to Valley Fair and stayed for 4 hours. She saw 4 lions, 3 deer and only 2 squirrels. How many animals did she see?"

Activity 6: Have children supply the appropriate questions for problems:

"While at the lake, Joan and Alan saw seven ducks. Four were floating and the rest were diving."

Activity 7: Give hints to help children recognize the strategy or process to be used:

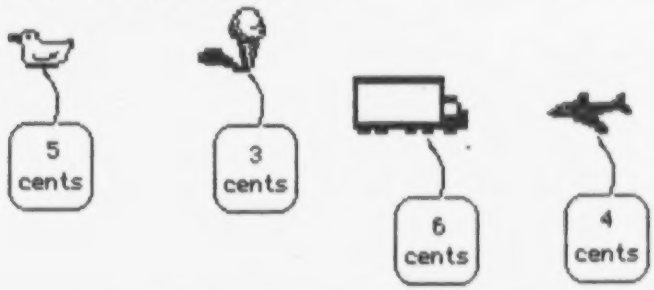
Example: "Anita's father took her to the track meet. Gloria won the race. Frances was last. Jean finished behind Janet. In what order did the girls finish?"

_____ _____ _____ _____

fastest slowest

Activity 8: Give children experience in deciding from choices which data to use.

"Tommy went to a flea market to buy some used toys. He spent 10 cents. Circle the toys he bought:




Activity 9: Take full advantage of a problem to get children to explore problems thoroughly, not just find one number answers.

Story 1: "On the Fourth of July, Sally and Joye watched the parade. They saw 4 trucks, 6 floats and a clown band. Sally drank 3 glasses of lemonade and Joyce drank 4 classes of lemonade.

1. What did Sally and Joyce see on the Fourth of July?
2. What was in the parade besides the clown band?
3. How many glasses of lemonade were drunk by Sally & Joyce?
4. Who drank the most lemonade?
5. What number sentence shows how many glasses of lemonade were drunk?

Story 2: On opening day of fishing, Fred went out in the boat with his father. They had 6 leeches and 3 hooks. Fred caught 2 perch on Friday and 3 perch on Saturday.

1. There were how many times as many leeches as hooks?
2. Fred caught fish on how many days?
3. How many perch did Fred cath?
4. Did Fred catch more perch on Friday or Saturday?
5. Draw a picture of Fred's catches on Friday and Saturday.

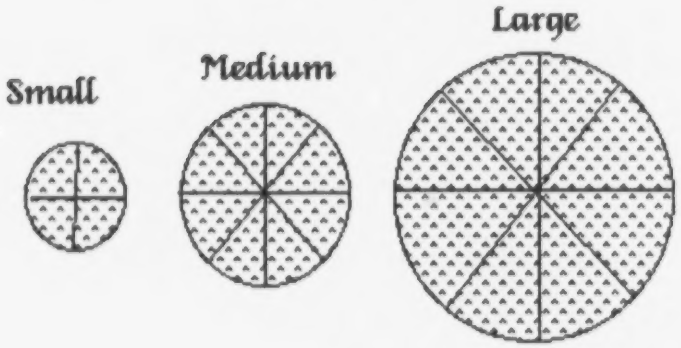
6. Which of these operations did you do?
   add  subtract  multiply  divide

Story 3: If John finished first in a race and Jackie fourth, how many runners are between John and Jackie?

Activity 10: Give children opportunity to interpret charts and graphs.

**AL'S PIZZA PARLOR**

**Sizes**

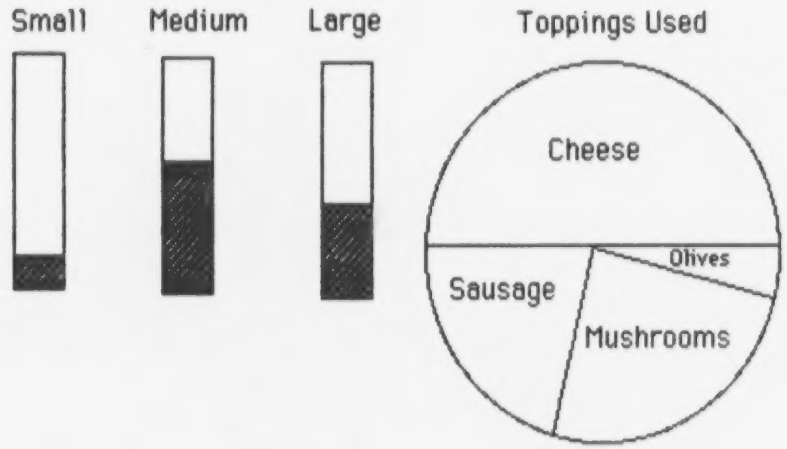



**Toppings**

1. Mushrooms
2. Olives
3. Sausage
4. Cheese

How many sizes does Al sell?
How many toppings does Al sell?
Which pizza serves the fewest people?
How many ways could you choose another topping to go along with cheese if you could have 2 toppings?

Yesterdays's Pizza Sales



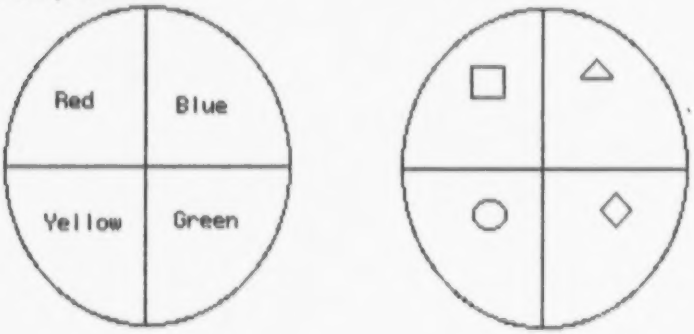

1. Which size was the best seller?
2. Which topping was the most popular?
3. Did the sausage and mushrooms together equal the cheese used?

THINKING

LESSON ONE: Give the children the four outfit sheet and 2 colors of crayons. Have them color the shirts and pants so no two are alike. The shirts and pants can be the same color or different colors. Give the children the six outfit sheet and 3 colors of crayons. Have them color the blouses and skirts so no two are alike. Blouses and skirts CANNOT be the same color.

LESSON TWO: Using Attribute blocks

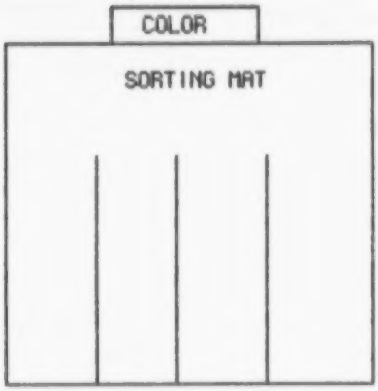
Introduction: Use transparent spinners for choosing color and shape. Spin each and ask the children which of the attribute blocks showing on the overhead (large only) should be selected.
Example:




Emphasize the block is red AND square, for example.

LESSON THREE

Introduction: Use overhead transparent Attribute Blocks and the 4 space sorting mat in transparency form. Put the "color" transparent label on the sorting mat as shown:




One at a time, show the children the blocks and ask them which bin they should go into. When all are sorted, ask the children to:

1. Count how many blocks are in each bin (8);
2. Describe the shapes in each bin;
3. Tell how many sizes of each shape there are.

Discuss as needed.

Extensions:

1. Use the 2 space sorting mat and the "size" label and ask the children to (1) count the blocks in each bin; (2) describe the colors in each bin; (3) tell how many shapes of each color there are.

2. Use the 4 space sorting bin and the "shape" label. Ask the children to (1) count the blocks in each bin; (2) describe the colors in each bin; (3) tell how many sizes of each shape there are.

LESSON FOUR

Use the 2 space sorting mat. Put all of the green pieces in one bin. Ask the children to use one word to describe the blocks in that bin. Ask them to use two words to describe the blocks in the other bin (NOT green). It may take some time to elicit this response and recognition that NOT indicates the absence of a property.

Extensions:

1. Repeat using a shape in one bin
2. Repeat using a size in one bin
3. Ask the children to choose a shape, color size to go in one

bin and have them describe what is in the other bin using two words

LESSON FIVE

Put the large green square and the small green square on the overhead. Ask the children to:

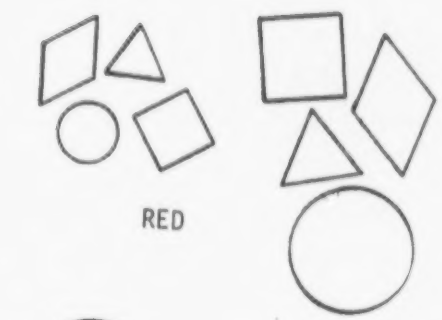
(1) Describe how these are alike. Try to elicit green AND square;

(2) Describe how these are different (size).

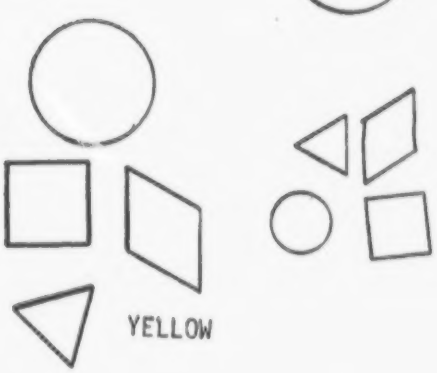
Extensions:

1. Using the properties color AND shape, color AND size, and shape AND size, sort the materials on the overhead so that those in a group are ALIKE on that pair of attributes.

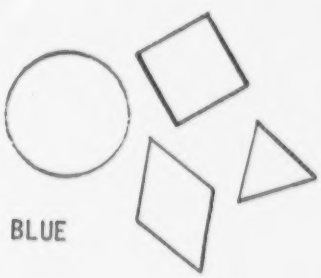
Example:



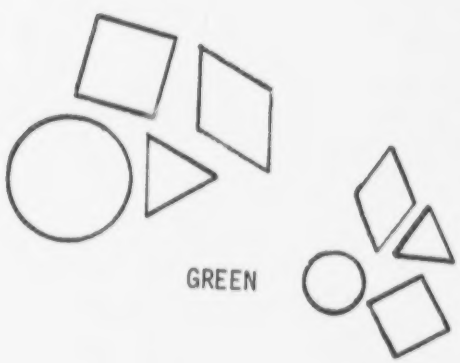
RED



YELLOW



BLUE

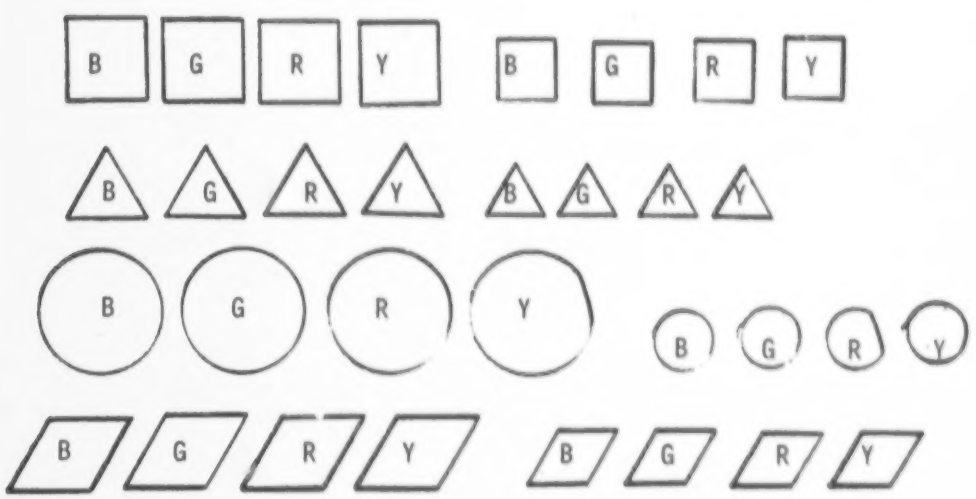


GREEN

Ask the children to (1) tell you how those within each group are ALIKE; (2) tell you how those within each group are DIFFERENT.

Repeat this for each of the two reamining pairs of attributes, asking the same questions.

LESSON SIX: *Twenty Questions*
Place all of the blocks in an orderly way on the overhead:




Tell the children you are thinking of a block and they are to try to learn which it is by asking questions that can be answered by "yes" or "no." You remove blocks in response to the questions.

Example:

LARGE RED

Is it small? NO - remove all small blocks
Is it green? NO - remove all green blocks
Does it have four sides? YES - remove all circles and triangles
Is it blue? NO - remove alll blue blocks
Is it square? YES - remove all diamonds
Is it red? YES - Leave the large red square there

LESSON SEVEN: *Who Am I?*
Example:

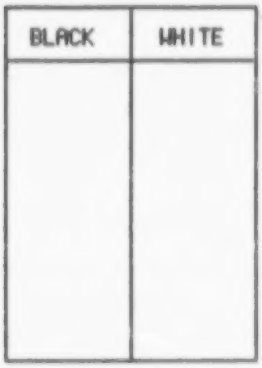
I am large
I am not red or green
I have no corners
I am not yellow
Who Am I?

LESSON EIGHT: *What Should I Put Down?*
Tell the children they are to help you make a line of blocks by telling you what to put down next.

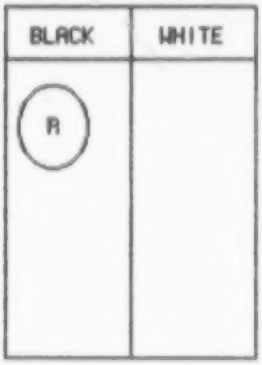
Example:
1. Start with a large green triangle
2. "I want to change shape only" - (large green square, circle or diamond can be used)
3. "I want to change color" - etc.

The changes specified will depend upon which blocks the children suggest.

LESSON NINE
Use an overhead transparency as shown and the large transparency Attribute Blocks:




The children have one black and one white cube each linked together. They are to hold up the link so the top cube shows which side a piece is to go to. Put a block on one side, i.e., the large red circle:




Show them a large red square and ask them to hold up the link to show where it goes. Some may want it with the circle since it is red. Others may want it in the WHITE column since it is not a circle. Use the majority opinion and place the block. Then take a large green square and ask where it goes. Then take a large green circle. Continue with the blocks until sorted.

Extensions:

1. Use only small blocks
2. Use only one color or shape blocks
3. Use all blocks

LEVEL ONE

GRAPHING

All of the suggestions for graphing in the Kindergarten lesson plans can be carried over into this level. Children will be more capable of making neater bar graphs with crayon pictographs. Making "pie" graphs requires an understanding of relationships between fractions that they may not have.

Things to graph:

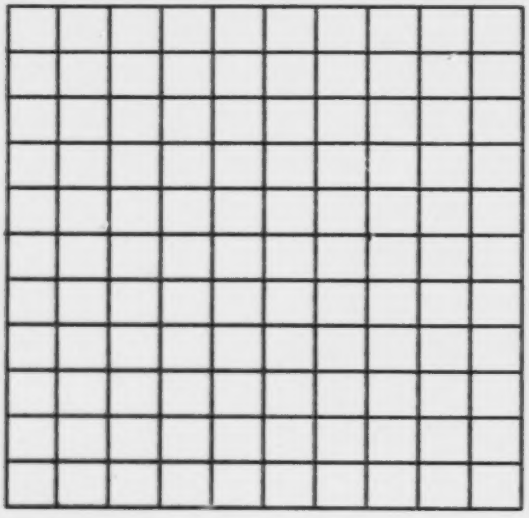
1. Sunny, cloudy, rainy, and snowy days
2. Days of the week on which someone in the class has a birthday, loses a tooth, etc.
3. The number of books read each week for 6 months
4. Juice for breakfast
5. Colors worn

LEVEL ONE

GRAPHING : (Continuous, throughout year)

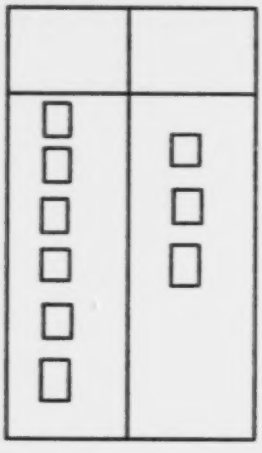
Background: There are several kinds of graphs that children at this level should make at various points during the year. These are listed below in order of development:

1. Real things - actually place objects selected for some reason on a "graph mat":



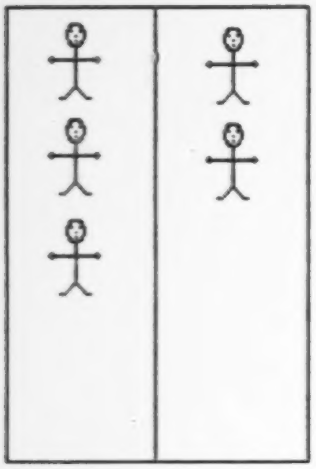
Made of plastic, oilcloth, butcher paper, etc.

2. Pictures of real things - children draw pictures of things, or use stickers and paste on a paper graph:

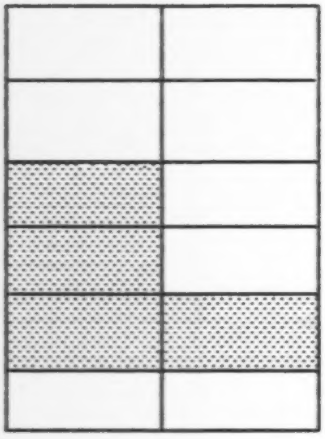


3. Children's names: Have children write their names on the graph, on "candles" or other features of the graph.

4. Stick figures to show children choosing an object, etc. - choices to be graphed:



5. Coloring in squares to represent choices:



Suggestions for graphs:

Yes or No questions
Do you like_____?
Are you right handed?
Do you wear glasses?
Are you a boy?
Are your eyes blue?
Did you walk to school?
Do you have a bank?
Do you eat _____?

Other two choice graphs:

Penny Toss - heads or tails
Did you pick ___ or ____?
Which is your favorite? _____or _____?
Does your name have (letter) or _____?
Do you feel good or bad today?
Did you wear mittens or gloves?

Three choice graphs:

Preferences of _____, _____ or _____
Shoes are plain, buckles, laces
Kind of clothing worn - slacks, skirt, dress; shirt, sweater, both.
One of _____, _____or _____ chosen.
Are you wearing something red, green or both?

There are several materials to use for preference or choice graphs:

Snacks
Colors
Books
Drinks
Clothing
Ages
Food type
TV shows or channels
Where live
Numbers, i.e. pockets
Feelings

There are several graph types:

Orientations:
top to bottom
bottom to top
vertical bars or
horizontal bars
Developmental Content
kind of day
highest temperature
teeth lost
birthdays
inches grown
monthly occurrences

Use several graphs related to periods in the school year:

<u>Fall</u>
snacks chosen
how reach school
same school last year?
teacher last year?
like school?
hobbies?
boys vs. girls
brothers and sisters?
seasonal fruit, e.g. apples, pears
leaves found
Tricks or Treat?
decorate house for _____?
weighing pumpkins - more or less than _____
popcorn - buttered or not?
favorite season song

<u>Christmas</u>
kind of Christmas tree
favorite Christmas character, e.g. Santa Claus, Frosty, Rudolph
letter to Santa?
stocking up?
where going on holiday?
favorite present?
pets?

Have some "situational" graphs and some that accumulate data throughout the year.

Have graphs that compare estimates with measurements: seeds in a pumpkins, M&M's in a bag, etc.

LEVEL ONE

GEOMETRY

Introduction:

LESSON ONE: Give a box of Geoblocks to each group of 10 children. Within each group, give each pair of children a worksheet with a shape on it. Have the children find as many blocks as they can so the block can "sit" on the worksheet and the face resting on the worksheet matches the outline on the worksheet.

LESSON TWO: Use the template to make enough sheets so you can cut out the triangles on the sheet, put these into an envelope and each child can have an envelope. Have the children find (1) all of the shapes that match exactly; (2) all f the shapes where 2 sides match, but not the third sides; (3) where just one side matches.

LESSON THREE: Give the children Pattern Blocks to use to make:

| Shape | From |
|---|---|
| Blue | Green |
| Red | Green |
| Yellow | Green |
| Red | Blue and Green |
| Yellow | Blue and Green |
| Yellow | Red |
| Yellow | Blue |
| Yellow | Red and Green |
| Yellow | Red, Blue and Green |

LESSON FOUR: Give the children Geoboards and a single rubber band. Have them make a shape using that band. Ask the children to group themselves:

(1) with others having the same shape, then
(2) with others having a shape with the same number of pins inside.

LESSON FIVE:
(1) Have the children find and trace all of the DIFFERENT triangular faces on the Geoblocks;
(2) Have the children find and trace all of the DIFFERENT rectangular faces on the Geoblocks;

These two activities can extend over several days at 15-20 minutes/day.

LESSON SIX: Have the children sort the Geoblocks into 2 groups using a property they decide upon. Follow up by sorting these into 3 groups.

LESSON SEVEN: Have the children sort the Geoblocks by the number of (1) faces they have; (2) number of "corners" they have.

General Activities:
1. Have a "shape for the day." Children are to find as many objects as they can with that shape on the object.

2. Use correct language with all geometric shapes, both solid and plane cube, cylinder, sphere, rectangular solid, prism, pyramid, etc.

3. Have children use Geoboards to make as many of the shapes that they find on Geoblocks, Attribute Blocks, Pattern Blocks as are possible on the Geoboard.

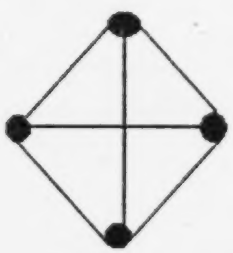
4. Have the children use LOGO commands to make as many shapes found on the manipulative materials available as they can.

5. Have children copy shapes you make of Pattern Blocks.

6. Give children toothpicks or matchsticks and play dough or clay. Ask them to make, in order:

6 "sticks" to make 2 triangles
5 "sticks" to make 2 triangles
6 "sticks" to make 4 triangles

The latter is:



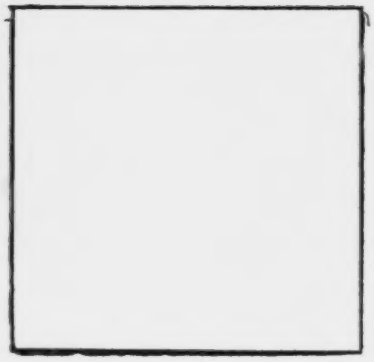
7. Using the template provided, cut out the pieces labelled A,B,C &D. Have the children rearrange them in a square.

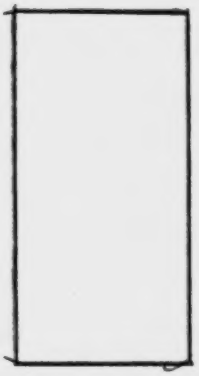
8. Obtain Tangrams and use the sheets provided for the children's use.

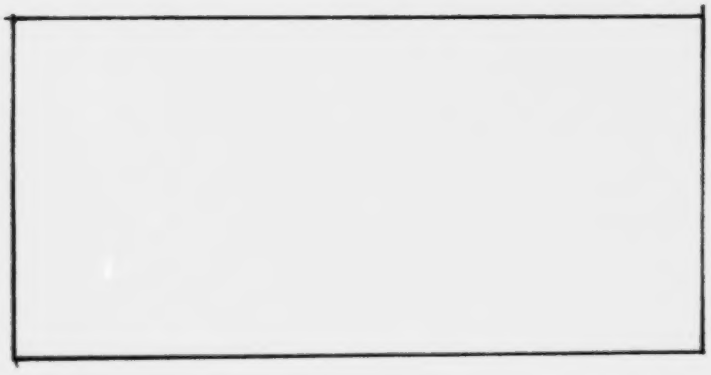
9. Obtain mirrors and have the children use the attached sheets to locate lines of SYMMETRY and to make mirror images.

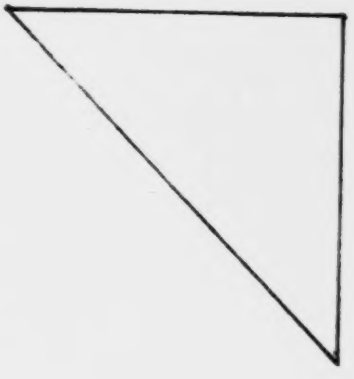
10. Fill an envelope for each child with six right triangles made from a 2 inch square. First have them use two of these to fit together into as many different shapes as they can. Then add one more triangle and have them do it again. Continue adding one more triangle until all six are in use.

TO THE TEACHER: This is ONE set of shapes children are to find on Geo Blocks by placing as many blocks as they can that have a face to match the shape.

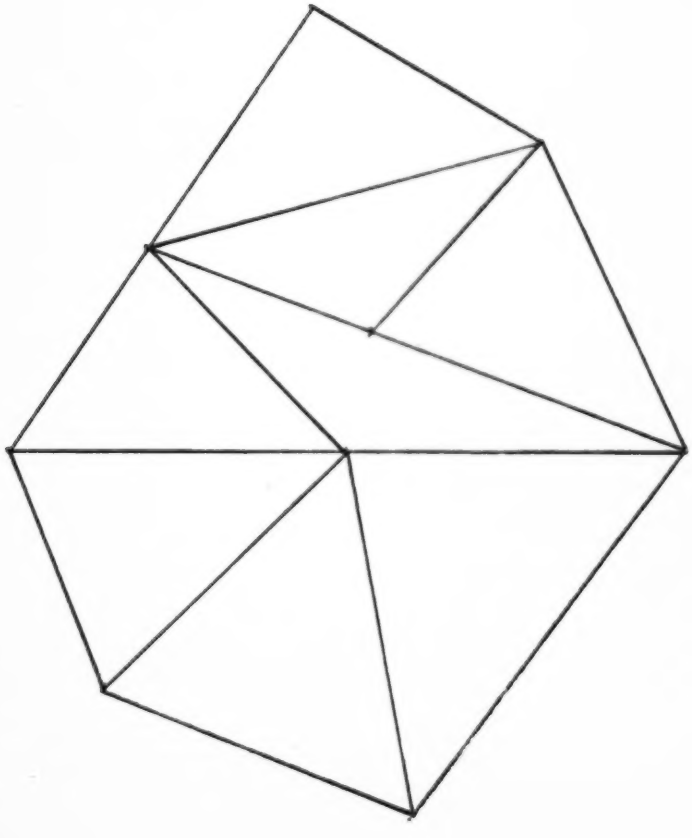


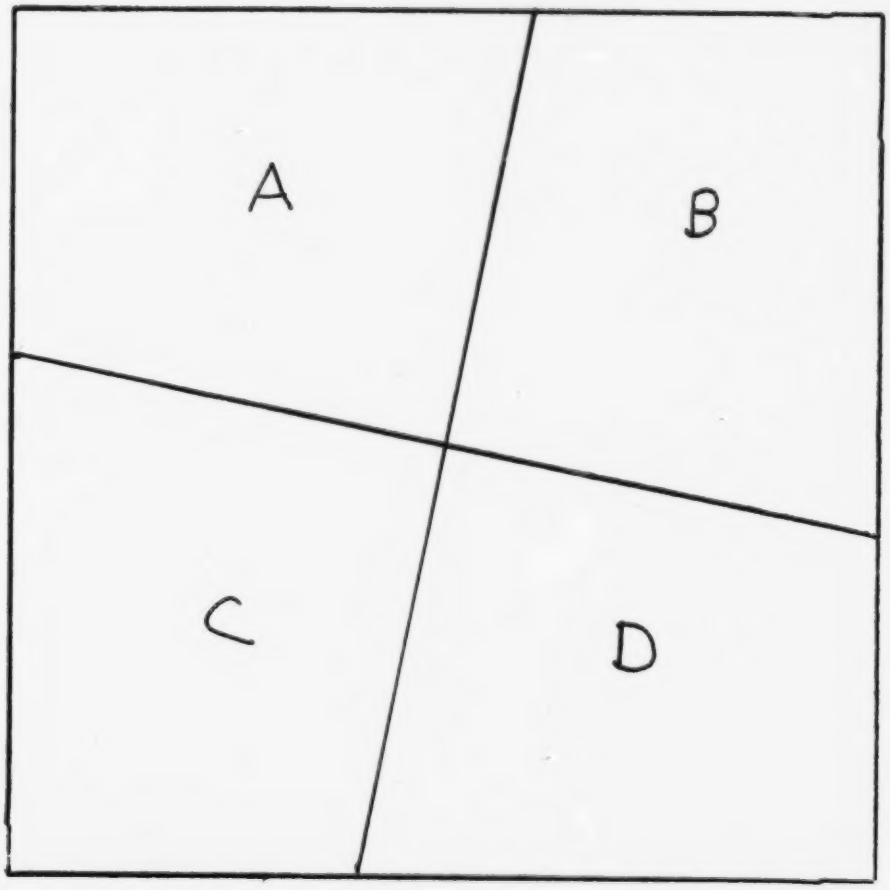


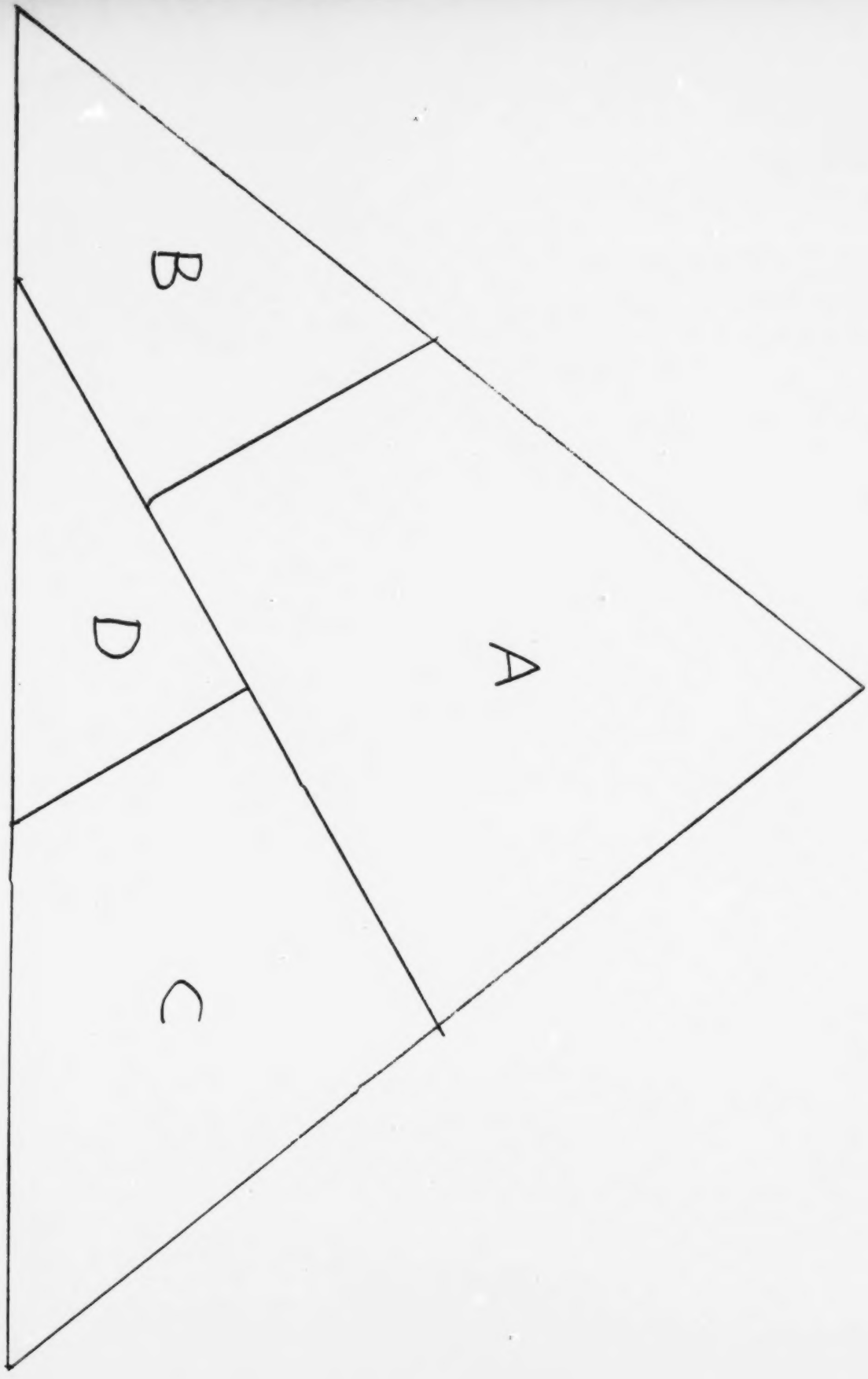




TO THE TEACHER: Make copies of the figure. Cut out all triangles in the figure to make a packet of triangles to use in "Geometry: LESSON 2."




A
B
C
D


B
A
D
C

LEVEL ONE

MEASUREMENT: Length

Background: Children should develop an understanding of what measurement IS before learning unit terminology or measuring mechanically with standard units. This means experience with different "units", comparing measurements with different "units", etc. The following suggested activities accomplish this for measurement of length. Easy to use "units" include paper clips, matchsticks, UNIFIX cubes and Cuisenaire rods.

Activity One:
Assign groups of two children to measure some object in the room using two different length "units", i.e. UNIFIX cubes and paper clips.

Have each group report (1) what was measured, and (2) the results in each "unit." Record these so the children can see them. Ask questions like:

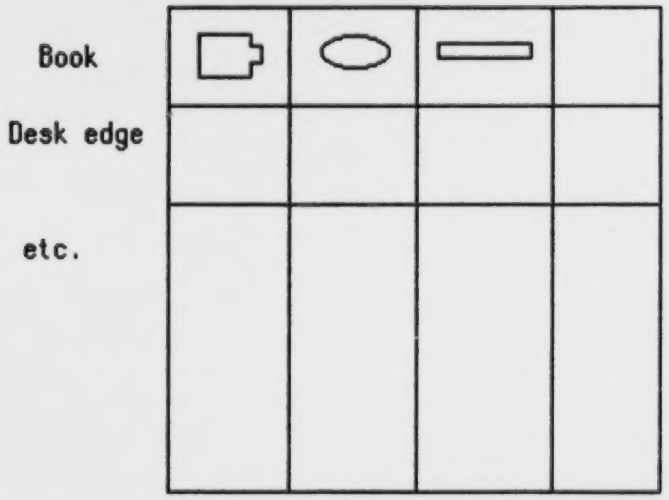
"Which is the longest length?"
"How do you know?"
"Why weren't the two measurements for each length the same number?"

Hold up two different lengths:

"Would I need more of this "unit" - or this "unit" to measure_____?"

The outcome of several such experiences and discussions should be realization that (1) to measure a given length requires MORE of a smaller unit and FEWER of a larger unit.

Make charts to compare lengths measured in several different units:



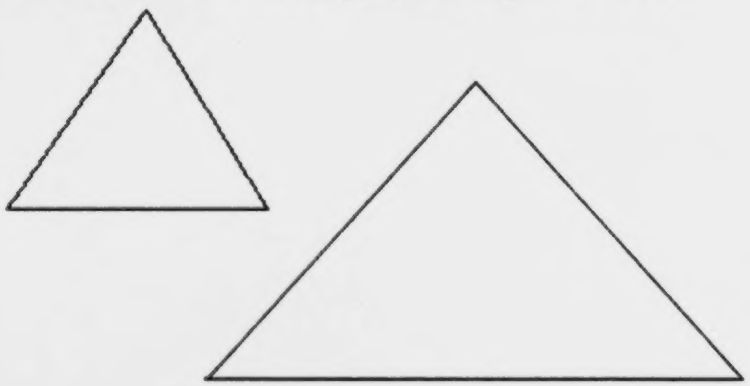

Extensions: Gradually introduce the idea of a "standard unit" such as an inch or meter. Need for such is based upon the need to have measurements made by different people in different places be understandable to people with whom they communicate.

LEVEL ONE

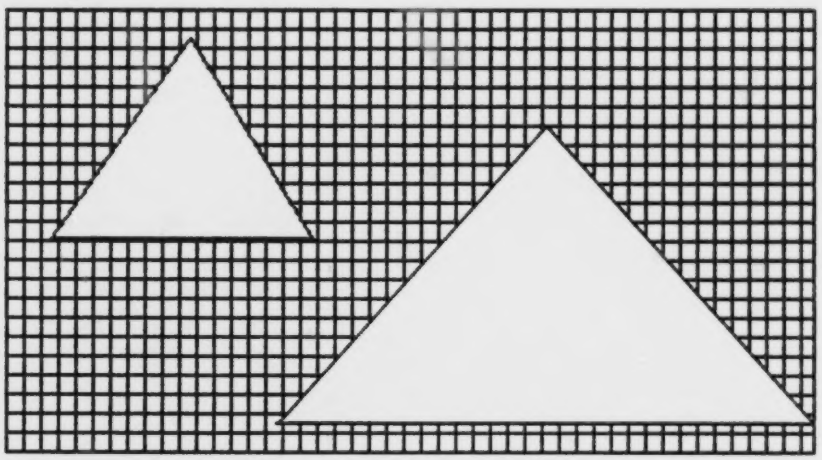
MEASUREMENT: Area

Background: If children have had some experience with Pattern Blocks and Geoboards, they probably realize different shapes "cover" more or less of a surface. They can then begin to consider ways to measure these differences.

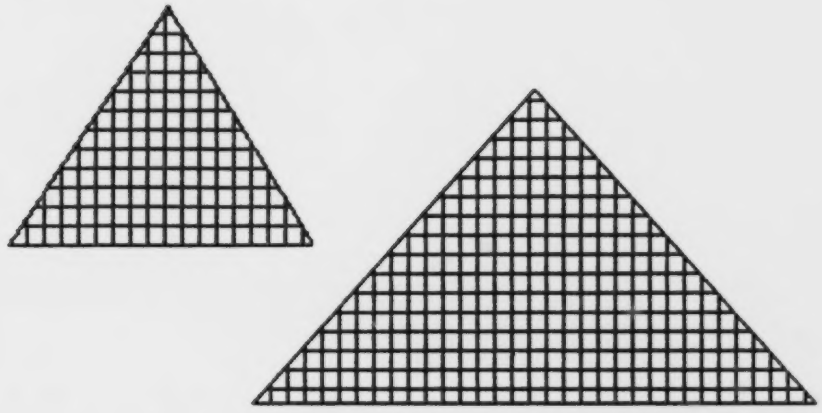
Introduction: Give the children 2 different sizes of the same shape that are made of a heavy paper or light cardboard:



Give them a sheet of 1" graph paper and have them trace each so that the tracings don't overlap:



Then ask them to count the "squares" inside each of the shapes. When they discover there are more inside the larger shape, point out that these "squares" measure how much each shape covers on how much room there is on each shape. This is the area of it and the squares measure that area:



Extended Activities: Have the children trace selected faces of objects such as Geoblocks, books, etc. on graph paper and compare their areas. They can cut out the tracings to have permanent "measured" shapes.

The question will come up of "part squares" on some tracings. The results are good if they count one as a square if more than half and ignore it if less than half of a square.

Have them make shapes on the Geoboard and then "measure" the area finding "squares" on the Geoboard.

Have them make unusual shapes and estimate areas of these using some "square" standard.

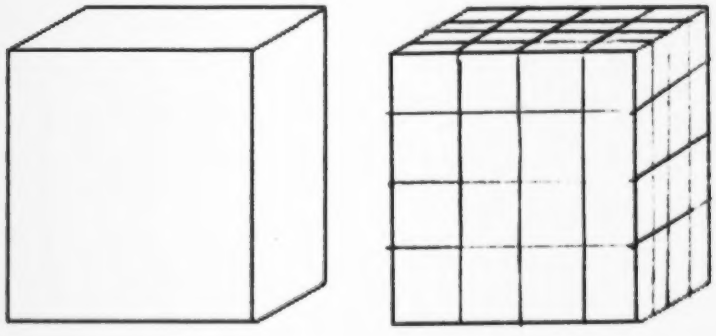
Have them compare the areas of the Pattern Blocks of faces of Geoblocks, of various objects in the room. Point out the "square" tiles on the floor and ceiling and have them give the measurements of these using these squares as "units."

LEVEL ONE

MEASUREMENT: Volume

Background: Children of this age are not likely to be conserving either area or volume, but working with volumes should not interfere with the development of those conserving behaviors.

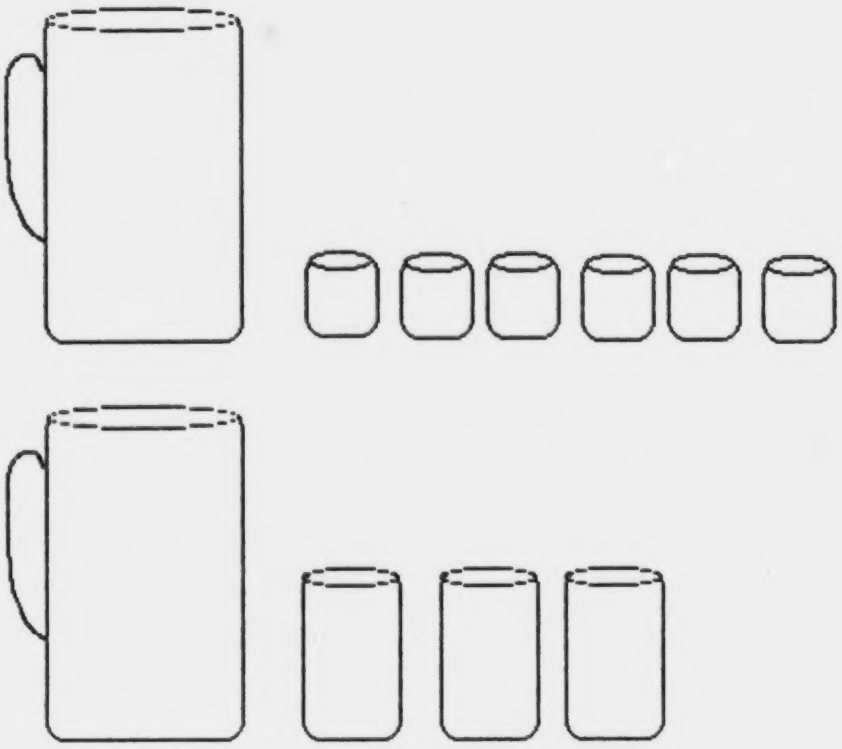
Introduction: Using Geoblocks, find larger cubes that are made up of some smaller cubes:



Point out that the smaller cubes are all the same and can show how much is in the cube, or how much a box made like the cube might hold.

Have the children find another cube that can be made from the same smaller cubes, but a different number of them. If you have mULTILINKS, now would be the time to use them to build larger cubes from smaller cubes.

Extended Activity: Find a larger container and use small plastic cups of the same size to measure the capacity of the larger container. Sand or water can be used. The small cup is the "unit" to measure how much the large container holds or its volume. This activity should be repeated with different size smaller cups so the children see that a unit is not natural, but set by agreement between people:

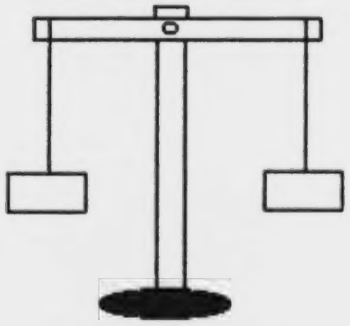


Here, too, the larger the unit used, the fewer needed to "measure" the container capacity.

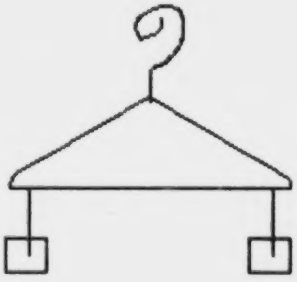
LEVEL ONE

MEASUREMENT: Weight

Background: Children love to use balances. You might use the OHAUS balance, or make balances from milk cartons and coat hangers. Don't use sophisticated pan balances such as are found in secondary science laboratories. Some examples are:



Made of 2 pieces of thin wood, like old meter sticks



Made of a coat hanger

Have them:

(1) estimate which of two items is heavier
balance them;
(2) weigh several small items using some "standard" such as washers, UNIFIX cubes, wooden cubes of the same size, etc., and then order these by weight.

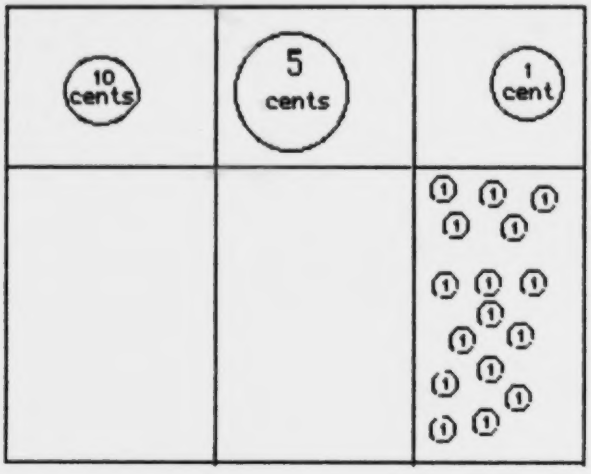
LEVEL ONE

MONEY

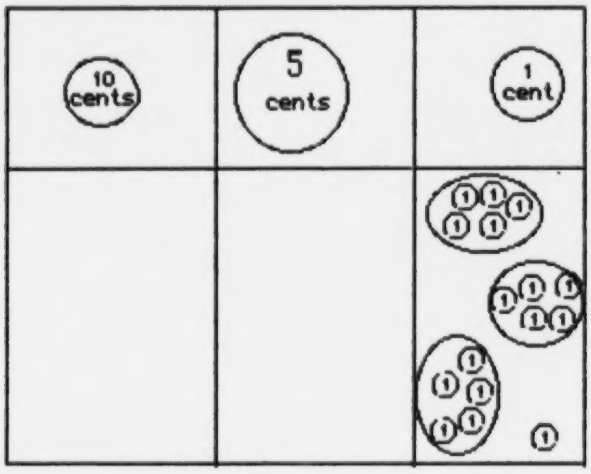
Background: In this grade, children should have introduction to the values of dimes and quarters. This is best done through an exchange process, using play money in the form of cardboard coins. Later children should have experienced evaluation, and mentally computing, collections of different coins.

LESSON ONE: Use a place value kind of mat with 3 columns and play coins.

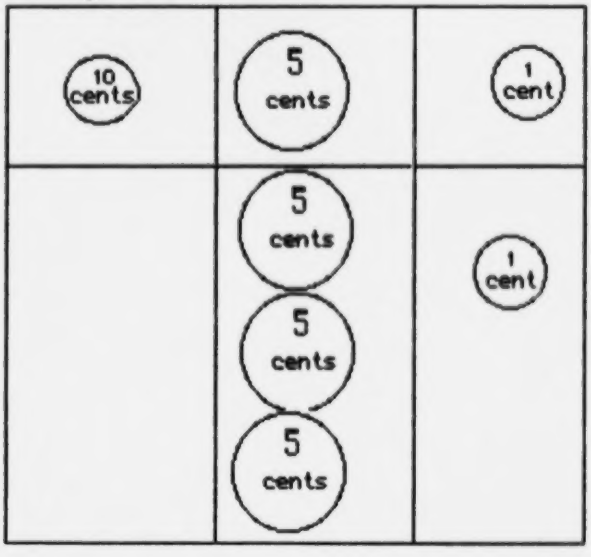
Introduce the lesson using an overhead version of the place value mat. Put a collection of sixteen or seventeen coins in the counter column:




Ask the children how many of the cents would be needed to exchange for one nickel. Most will know five, so group the cents into fives:

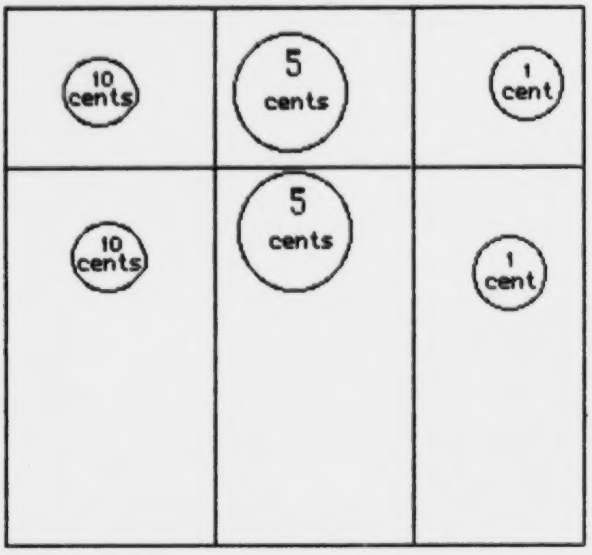



Exchange these for nickels:




Then ask if any further exchange could be made. Some children will suggest nickels for dimes. Point out that TWO nickels exchange for one dime since BOTH represent TEN (two fives) cents.

Make the exchange:




Note that one dime + one nickel + one cent
Ten + Five + ONE

is the same as the sixteen cents at the beginning.

Give the children different numbers of cents to start with and have them make the exchanges until not more are possible.

This exchange activity should be repeated often enough later in the year so that children easily recognize the TOTAL cents value of combinations of small numbers of dimes, nickels and cents.

LESSON TWO: The exchange activity should work backwards also. Start with a small number of dimes and exchange first for nickels, then for cents - using the same approach and materials.

LESSON THREE: When children have mastered cents, nickels and dimes as to exchange and recognition of value of a total collection, introduce quarters using a four column mat.

The same activities of (1) making the fewest coin representation for a given amount in cents, and (2) converting a given collection of coins into cents should be done.

TO: THE TEACHER

FROM: A. DEAN HENDRICKSON

Attached is something you can reproduce and send home to parents to encourage them to support what you are doing in the classroom.

# MATHEMATICS IN THE HOME

A. Dean Hendrickson, University of Minnesota-Duluth

## Introduction:

Mathematics and the use of mathematical thinking is much more than what has been traditional school arithmetic. The arithmetic of whole numbers, fractions and decimals constitutes no more than 10-15% of the mathematics we use throughout our lives. Much of the mathematical reasoning we use can be developed and experienced out of school, particularly in the home. Some of these suggestions may seem remote from the arithmetic you remember, but they will involve children in the THINKING essential to both the learning and use of mathematics in everyday life.

## Pre-Mathematical Thinking:

Before a child can understand school mathematics, certain ways of thinking and skills must be available for use. These are continuously used throughout learning of mathematics, but particularly elementary school mathematics. These include: counting, comparing, ordering, using patterns, using grouped material, using language and establishing relations and relationships. Needed experience with these can be obtained around the home. Before describing things to do with children at home to help them with their school mathematics, here are some "golden rules" based upon research and experience with learning children.

1. You must not force children since this has negative effects, such as turning them away from doing things or from you. A child learns when ready, curious, and needing to make sense of something. This goes in spades for drill on memorizing so-called "basic facts."

2. Give children positive things to do when time is available, especially these things they can do and enjoy doing. Don't ask for things beyond the child's capacity to do.

3. Give lots of praise and encouragement. If what the child does or says doesn't seem to make sense to you, don't criticize or correct. Ask questions that might lead the child to consider it in a different way.

4. Don't look for day-to-day progress or change or for immediate results. Just as with many other things, such as walking or talking, a child may seem to be making no headway and then suddenly, it's all there. Children develop in spurts and unevenly, and have long plateaus where nothing seems to be happening. That's normal and accept it. There is probably a lot going on below the surface.

5. Don't compare yours with other children. Everyone is different - thank goodness!

6. Don't worry if a particular skill, such as using language, is

coming along more slowly than you'd like or than brother John's did. Somehow most of them seem quite a lot alike by the time they are 12 or so.

Words:

A number of words commonly used in mathematics and related to teaching mathematics should be used often outside of school as well. Some examples are *some-more, a lot, more than, less than, large, small, many, few, some as, different, alike, all, same, not, left, right, ahead of, behind, above, below, front, back, long, short.*

In addition to words associated with comparing, grouping and space, the number words are important. Children must know the counting words, but even more than that, they must see the pattern in the use of counting words. The cardial words like *first, second, third,* et. are also important. Use of these words around home helps children to count objects correctly and to identify position of things in ordered arrangements.

Comparing:

Have children compare things as to size, length, are and volume whenever possible. "Which glass has more?" "Which box holds more?" "which of these is heavier? heaviest?" "Put these sticks in order of length." "Arrange the silverware so the longest is farthest from the plate and the shortest is nearest the plate." Questions like these should be frequent. They should involve different kinds of things both indoors and outdoors. Combine these with questions that make the children estimate measurements of distance and height such as "Which do you think is as high as the shed, A or B?"

Comparing of quantity leads to better understanding of number and number relationships. "Are there more chairs or lamps in this room?" "Are there more cups or teaspoons on the table?" "Have we go more red roofs or green roofs on our street?" "Put enough table knives on the table so that there are as many knives as forks." "Do you have more boys or girls in your class?" These can be asked when out walking, riding in the car, watching TV or sitting in the boat. Ask children to do things that will make one group as large as another frequently All such activity helps children build number relations into their deeper understandings, instead of as memorized associations that have no meaning - like names and dates you once memorized to pass a history test!

Ordering things that can be counted is important. Bead stringing activities are good for young children. "String some beads so the third bead is red and the fourth bead is blue." "Make a string so every other one is green," etc.

Ordering things that have lengths, areas and volumes extends comparing beyond two things. Have children place three sticks of different lengths in order from shortest to longest; place three pieces of paper of different areas into orders; place three different sized cans of jars into order. Gradually extend the number of things to more than three for these activities.

of things - knives, forks, chairs, chair legs and table legs, buttons, marbles, pieces of candy, etc., help the child with what the school is doing.

2. Join together several groups of the same size into a larger group. Rows of pennies can be arranged into an array like this and can then be looked at a different way to see 5 groups of 6 pennies:

Both lead to a total of 30 in the array. Do this in a row at a time, having the child tell you how many are there all together each time. Separate and take apart such arrays row by row and see what is left each time. Do this with different kinds of things, different size rows and different total numbers of things. Clothes pins, ceramic tiles, beans, corn are all good for this.

3. Join together groups of different size, such as seven things with five things. Have the child describe what is happening in words. Have the child add to one group of things enough to make it the same size as another larger group. Have the child make equal two unequal size groups without adding anything more to the collection. "Here are a group of 15 clothes pins and one of 7 clothes pins. Do something so you have two equal groups."

4. Give the child large amounts - in the 20's or 30's of things to:

   a) make several groups of a given size from. Some numbers should make these smaller groups an even number of times and some should have some left that is not enough to make another of the smaller group.

   b) make a certain number of groups that will all have just as many in them.

   Examples:

   "Put these 30 beans into 6 cups, so each cup has just as many. How many are in each cu?"

   "Put these 43 beans, six at a time into cups. How many cups did you use?" "What should e done with what is left over?" "When do you have some left over?" "When don't you have anything left over?"

When you do for walks, have the children compare, add together, etc., things along the way. Do the same in the car, the supermarket, in the drugstore. "How many are there on the top shelf?" "How many re on the bottom shelf?" "How many are there on the top and bottom shelves together?"

Have the child do as much adding, subtracting, multiplying and dividing of this kind - always as related to things - as you can. <u>DON'T</u> try to drill your child on "addition" facts or "multiplication" facts. Let the child learn these in due time

through the school activities and those you do at home as described here. DON'T have the child write number things - the school will do this. Accept verbal answers and descriptions. Get in the habit of asking your child why certain answers are given and LISTEN.

SOME FINAL HINTS:

1. Have your children count things as much as possible.

2. Ask children simple addition, subtraction questions about REAL things in the surroundings to give practice in mental arithmetic.

3. Play card games that require mathematics or related things like WAR, OLD MAID, CRIBBAGE, RUMMY (regular or gin).

4. Give thinking games for holiday gifts - CONCENTRATION, HUSKER DU, etc.

5. Get a Little Professor or some similar calculator-based program to give mental arithmetic practice.

6. Cheap mathematics games can be bought at Target, Woolworths, etc. Some examples are COVER UP, HEADS UP, SCORE FOUR, TUF, APOLLO, etc.

7. Give your child a simple four function calculator and let him or her fool around with it.

8. Encourage block play and building, sand play, making birdhouses, etc.

9. Key words are COMPARING, COUNTING, PATTERNS, COMBINING (groups), SEPARATING (large groups into smaller groups)

10. Point out mathematics wherever it is in the surroundings. Children must realize mathematics is:

    a. easy to learn
    b. useful
    c. fun

Ask frequent questions about the ordering of events as to which happens first, second...last, etc. Connect these with time estimations, "How many minutes ago do you think this happened? How many days?" etc.

Counting:

Children should keep extending their memorized sequence of counting words. This is important. But being able to say the words in right order does not mean they can count things. They need much practice at this. Have them count everything around the house that is countable - the chairs, tables, legs on chairs; the tiles on the floor, in the ceiling; the number of windows in a room; the silverware in the drawer; the cans on the shelf; the pieces of wood in the woodpile; the telephone poles going by, etc. The more they count, the better able they are to count. When they are pretty good at counting forward, have them do some counting back. For example, start with 20 clothespins. One at at time put one into a can and count aloud those that are left as each one is removed from the pile.

Patterns:

Have children look for patterns - in the carpet, in the ceiling, in wallpaper, in the drapes, on the bedspreads. Patterns of shape, or color, or sound are all important. Beads can be strung in patterns. Collections of bottle caps, old keys, buttons, screws, nuts and bolts, and similar "junk" can be put into patterns. Ask children what would come next in a pattern, or what would go where something is missing in a pattern.

Number:

Help your child learn number size by having him see the same number, such as five, in many different arrangements and materials. Playing cards can be sorted into those all having the same number. Mixed groups of say, five marbles, three buttons, three keys, six spoons, can be used. "Find me the material there are five of," etc. Put some number, seven for example, of beads or marbles into three or four different shaped glass jars, "Find a jar with seven in it." "Find another." Put the same number of one kind of thing in one jar and another kind in a second jar, etc., and do the same kind of thing. Involve the child with numbers in as many different ways, with as many different kinds of material, and as many different sizes as possible. Gradually increase the number size as the child seems able to easily handle smaller numbers.

Using Numbers:

Comparing groups with number property; combining such groups; separating larger groups into smaller groups of a given size or into equal size groups - all of these activities help children to understand when each of the four arithmetic operations are used.

Some examples of things to do in the home of this kind are:

1. Compare two different sized groups in several ways. "How many more are there in this group than in that group?" "This group has how many fewer than that group?" "How many times as many are there here as there?" These kinds of questions used with groups of all kinds